# GOVERNMENT COLLEGE FOR GIRLS, LUDHIANA

## DEPARTMENT OF MATHEMATICS

E-CONTENT
for the
CLASS B.A/B.Sc 3rd
LINEAR ALGEBRA

# LINEAR TRANSFORMATION and MATRIX TRANSFORMATION

Today, we will explore the concepts of linear transformations and matrices and their applications.

# Linear Transformations

- A linear transformation is a function that maps vectors from one vector space to another while preserving the properties of linearity.
- It can be represented by a matrix multiplication.
- Examples of linear transformations include rotations, Reflection, scaling, and shearing.

# DEFINITION:-

In Linear Algebra, a transformation between two vector spaces is a rule that assigns a vector in a one space to another vector space. These transformations are satisfied a certain property in addition and scalar multiplication. This transformation relates "image'' and range'' and this makes these transformation different from other transformations.

IN OTHER WORDS ,
If V(F) and W(F) are two vector spaces, then a mapping T from V to W i.e.; $T: V \rightarrow W$ is said to be a Linear Transformation if and only if

1. $T(x+y)=T(x)+T(y)$, where $x,y \in V$
2. $T(\alpha x)=\alpha T(x)$ ,where $x \in V$ and $\alpha \in F$

OR
combining both the conditions we've one property i.e
$T(\alpha x+\beta y)=\alpha T(x)+\beta T(y)$

e.g.

Let's consider the transformation T: V -> W, where T is defined as

$T(x, y) = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$ Here, θ is a fixed angle, and $(x, y) \in V$

The transformation T rotates the vector (x, y) counter clockwise by an angle of θ.now to prove that T is a linear transformation, we need to check that it satisfies two properties as we discussed above (as x & y are given in equation so take another one variables like u, v.

1.$T(u + v) = T(x1 + x2, y1 + y2)$

$= ((x1 + x2)\cos\theta - (y1 + y2)\sin\theta\ ,$

$(x1 + x2)\sin\theta + (y1 + y2)\cos\theta)$

$= (x1\cos\theta - y1\sin\theta, x1\sin\theta + y1\cos\theta)$

$+(x2\cos\theta - y2\sin\theta, x2\sin\theta + y2\cos\theta)$

$= T(x1, y1) + T(x2, y2)$

$= T(u) + T(v)$

2. $T(\alpha u) = T(\alpha x, \alpha y)$

$= (\alpha\, x\cos\Theta - \alpha\, y\sin\Theta,\ \alpha\, x\sin\Theta + \alpha\, y\cos\Theta)$

$= \alpha(x\cos\Theta - y\sin\Theta,\ x\sin\Theta + y\cos\Theta)$

$= \alpha\, T(u)$

Therefore, T is a linear transformation.

Try these by yourself that the given transformations are linear or not.

Let T be a transformation defined by

1. $T: R^3 \rightarrow R^2$ defined by $T(x,y,z)=(x+y, x-z)$ for all $x(,y,z) \in R^3$

2. $T : R^2 \rightarrow R^2$, defined by $T(x,y)=(x+y,y)$ ; $(x,y) \in R^2$

3. $T : R^2 \rightarrow R^2$ defined by $T(x, y) = (x^2, y)$.

# Matrices

- Matrices are used to represent linear transformations.
- A matrix is a rectangular array of numbers, where each number is called an element.
- The size of a matrix is determined by the number of rows and columns it has. E.g if a matrix is of order n x m.it means it has  n rows and m number of columns.

# Matrix Tranformation

- DEFINITION:-
- Let T : $R^n \rightarrow R^m$ defined by $T(x) = Ax$ , $x \in R^n$
  and A is some $m \times n$ matrix. Then for each $x \in R^n$ $T(x)$ is computed by Ax, where A is an $m \times n$ matrix

i.e, $T(x) \varepsilon R^n$ . Observe that the domain of T is $R^n$, when A has n columns and the co-domain of T is $R^m$ when each column of A has m entries. The range of T is the set of all linear combinations of the columns of A, because each image w = T(x) of x is of the form Ax. This mapping is called matrix transformation columns of A, because each image w = T(x) of x is of the form Ax. This mapping is called matrix transformation.

# Matrix Representation of Linear Transformations

- Each column of the matrix represents the image of a basis vector.
- Example:-

Scaling Transformation

A= [2 0; 0 3] ,

Here Image of [1 0] = [2 0]

and the Image of [0 1] = [0 3].

## Scaling followed by Rotation•

A=[2 0; 0 3] * [cosθ -sinθ; sinθ cosθ]

# Applications of Linear Transformations and Matrices

=>Linear transformations and matrices have various applications in different fields.

1.In computer graphics, they are used for 2D and 3D transformations, such as rotating and scaling objects.

2.In physics, matrices are used to describe physical systems and solve equations.

3.In data analysis, matrices are used for dimensionality reduction and feature extraction.

4.The goal is to transform the vectors in a way that maintains these properties and possibly changes their orientation, shape, or size.

5. Linear transformations are commonly used in various fields such as mathematics, engineering to model and analyze different phenomena.

6. In economics, linear transformations are used to model relationships between variables in linear regression models.

# Conclusion

#Linear transformations and matrices are powerful tools for representing and manipulating data .

#They have numerous applications in various fields, including computer graphics, machine learning, physics, and data analysis .

#Understanding these concepts is essential for anyone working with vectors and transformations.

Thank you