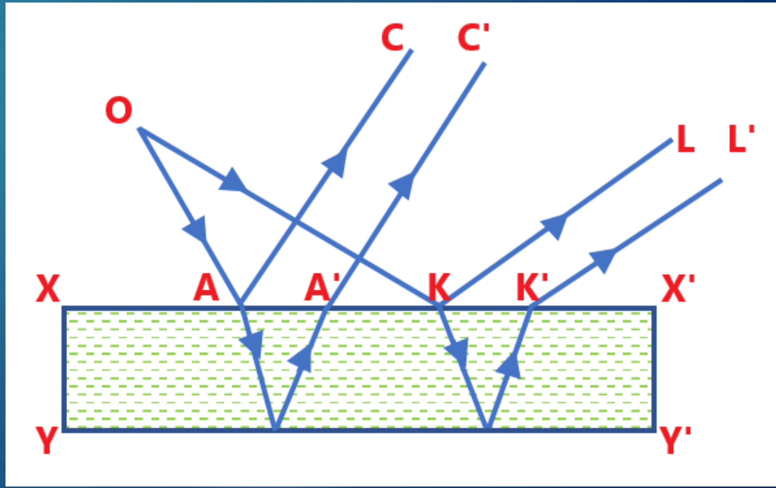
INTERFERENCE BY DIVISION OF AMPLITUDE

Formation of colours in thin films

When thin films are seen in monochromatic light, the minima are dark and maxima are bright. But when interference in thin films is observed in white light, maxima and minima formed are colored because condition for maxima and minima depends upon wavelength of light. That is why we see colors in soap bubbles and thin layers of oil floating on the surface of water etc.

Need for an extended source for interference by division of amplitude

The interference in thin films can be observed without using lens, because the eye lens can form real image of interference pattern on retina. With a point source of light we get different rays which are incident on the film at different angles. Therefore reflected rays goes in different directions and all the rays will not be able to enter in eye as shown in fig.

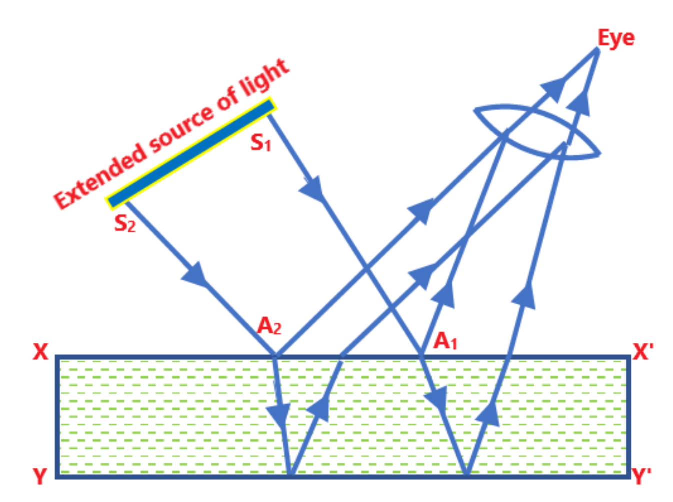


INTERFERENCE BY DIVISION OF AMPLITUDE

Need for an extended source continued….

Therefore we are able to see only a very small portion of interference pattern. To see the interference pattern in different parts of the film, the eye need to be moved side ways. Thus a point source is not suitable for observing interference in whole thin films simultaneously.

But if extended source of light is used, the light from different parts of source will strike the surface of thin film at different angles and after reflection from the whole surface of thin film the light manage to enter in the eye as shown in fig. As a result, the interference pattern can be observed on whole of the film simultaneously.



INTERFERENCE BY DIVISION OF AMPLITUDE

Haidinger fringes-Fringes of equal inclination

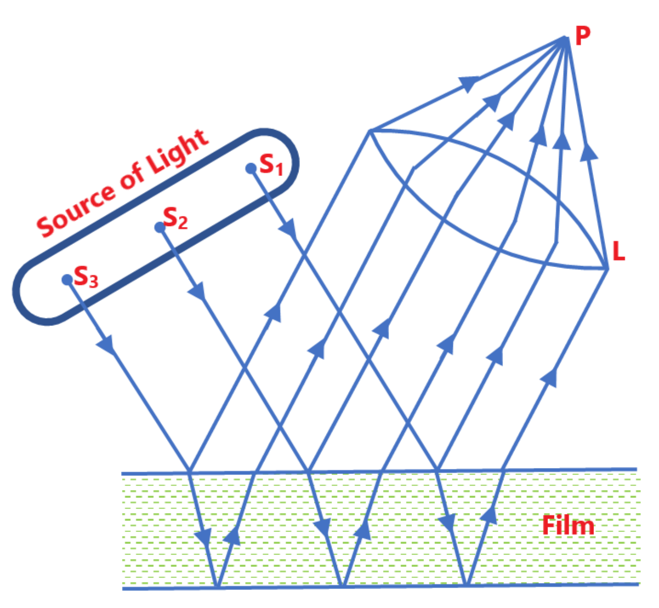


Fig. shows an extended source of light having S_1, S_2 and S_3 as three points on it. The rays of light originating from S_1, S_2 and S_3 and parallel to each other, are incident on a thin film and the corresponding beams reflected from its upper and lower surfaces are focused to a point P by a lens (may be eye lens).The fringes so formed are called Haidinger fringes.

Condition for observing interference pattern

The points S_1, S_2, and S_3 are independent sources. These are non-coherent sources. Hence a sustained interference is produced only if the light beam originating from each point after division of amplitude again passes through the lens. If the aperture of lens is so small that this does not happen, the interference pattern is not seen. If the film thickness is large, then there will be large separation between the beams and both of them may not pass through lens.

INTERFERENCE BY DIVISION OF AMPLITUDE

Haidinger fringes continued….

Localized fringes

Interference pattern cannot be obtained on screen without lens. We are able to see the pattern because of eye lens. Thus they are virtual fringes because light rays are parallel to each other and fringes are located at infinity. Hence they are also called localized fringes because they are localized as virtual fringes at infinity.

Note: Fringes obtained in Young's double slit experiment are non-localized because they are formed everywhere. We can place the screen at any distance.

Fringes of equal inclination

Since these fringes are formed by parallel incident beams from different points on an extended source, they are also called the fringes of equal inclination

INTERFERENCE BY DIVISION OF AMPLITUDE

Fizeau fringes-Fringes of equal thickness

In thin films path difference is given by: $\Delta = 2nt\cos r$

If the thin film has varying thickness(t), the path difference will vary even without varying the angle of incidence or angle of refraction (r). Hence the fringe pattern will be associated with a particular thickness of the film. For this reason, they are called fringes of equal thickness. They are also known as *Fizeau fringes*.

In fig. (1), we have wedge shaped film of varying thickness illuminated normally.

These are also localized fringes because they appear to be localized in the film as shown in fig. (2)

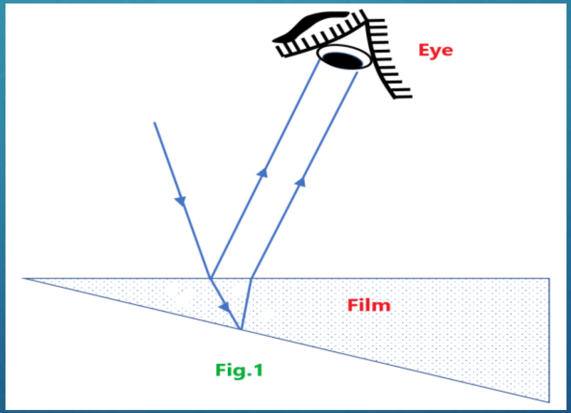


Fig.1

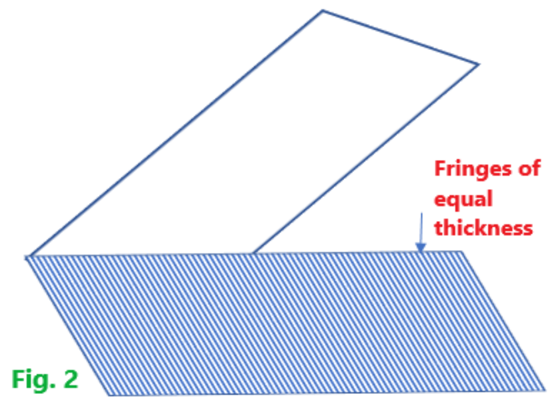


Fig. 2

INTERFERENCE BY DIVISION OF AMPLITUDE

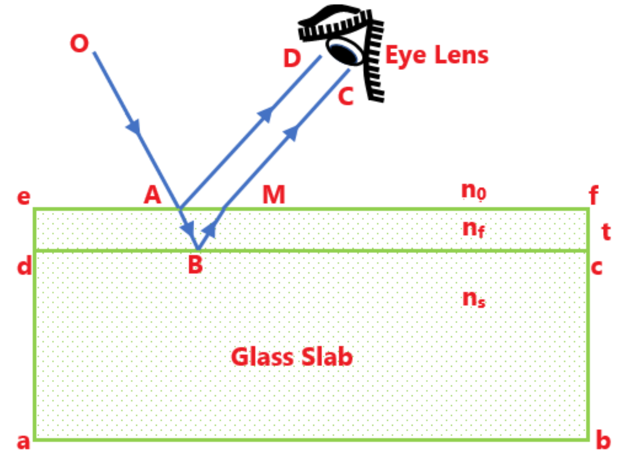
NON-REFLECTING FILMS

A thin layer of transparent medium coated on the surface of lens or slab, such that the reflection of light incident on it is minimized is called non-reflecting film.

Thickness of non-reflecting film

The non-reflecting films are based upon the principle of reflection by division of amplitude. Fig. shows a slab abcd having material of refractive index n_s on which a thin layer cdef of some different transparent material of refractive index n_f is deposited. n_0 is the refractive index of medium from which light is incident.

The incident ray of light OA splits into two parts at A. The reflected ray goes towards AD and the refracted part is internally reflected at B and then proceeds along BHC. Two rays AD and HC interfere when they superimpose. Let thickness of film cdef be 't'. Suppose $n_s > n_f > n_0$. In this case, both the reflections at A as well



INTERFERENCE BY DIVISION OF AMPLITUDE

NON-REFLECTING FILMS CONTINUED….

as at B occur at denser medium and due to reflection the phase difference introduced between AD and HC is 2π, which is equivalent to zero phase difference. Hence net path difference between the rays AD and HC will be:

$$\Delta = 2nt\cos r$$

Where n is the refractive index of the film w.r.t. the medium from which the ray OA is incident. i.e. $n = \frac{n_f}{n_0}$, Hence:

$$\Delta = 2\frac{n_f}{n_0}t\cos r$$

If light is incident from air, then $n_0 = 1$, therefore: $\Delta = 2n_f t\cos r$

For normal incidence, both angle of incidence and angle of refraction are zero

$$\Delta = 2n_f t \qquad \text{.....(54)}$$

INTERFERENCE BY DIVISION OF AMPLITUDE

NON-REFLECTING FILMS CONTINUED….

The rays AD and HC will interfere destructively if: $\Delta = (2p+1)\frac{\lambda_0}{2}$

Where λ_0 is the wavelength of light in medium from which it is incident.

The minimum value of path difference will be for p=0, hence minimum path difference is: $\Delta = \frac{\lambda_0}{2}$

Substituting in (54), we get: $2n_f t = \frac{\lambda_0}{2}$ …..(55)

If λ_f is wavelength of light in medium, then: $\lambda_0 = n_f \lambda_f$

Hence equation (55) becomes: $2n_f t = \frac{n_f \lambda_f}{2}$ or $2t = \frac{\lambda_f}{2}$

or $t = \frac{\lambda_f}{4}$ …..(56)

This gives the thickness of film to be deposited on material.

INTERFERENCE BY DIVISION OF AMPLITUDE

NON-REFLECTING FILMS CONTINUED….

Refractive index of non-reflecting film

If the refractive index of the medium from which reflection takes place is 'n', then the coefficient of reflection of amplitude of electric vector of light is given by: $r_E = \frac{1-n}{1+n}$

Now for the reflection at the interface of media of refractive indices n_0 and n_f , we have: $n = \frac{n_f}{n_0}$, therefore:

$$r_{E_1} = \frac{1-n_f/n_0}{1+n_f/n_0}$$

Again for the reflection at the interface of media of refractive indices n_s and n_f, we have: $n = \frac{n_s}{n_f}$, therefore:

$$r_{E_2} = \frac{1-n_s/n_f}{1+n_s/n_f}$$

INTERFERENCE BY DIVISION OF AMPLITUDE

NON-REFLECTING FILMS CONTINUED….

Interference will be completely destructive if amplitudes of two waves are same

i.e. $r_{E_1} = r_{E_2}$ or $\frac{1-n_f/n_0}{1+n_f/n_0} = \frac{1-n_s/n_f}{1+n_s/n_f}$ or $\frac{n_0-n_f}{n_0+n_f} = \frac{n_f-n_s}{n_f+n_s}$

or $n_0 n_f + n_0 n_s - n_f^2 - n_s n_f = n_0 n_f - n_0 n_s + n_f^2 - n_f n_s$

or $2n_f^2 = 2n_0 n_s$ or $n_f = \sqrt{n_0 n_s}$ …..(57)

For air $n_0 = 1$, Therefore: $n_f = \sqrt{n_s}$

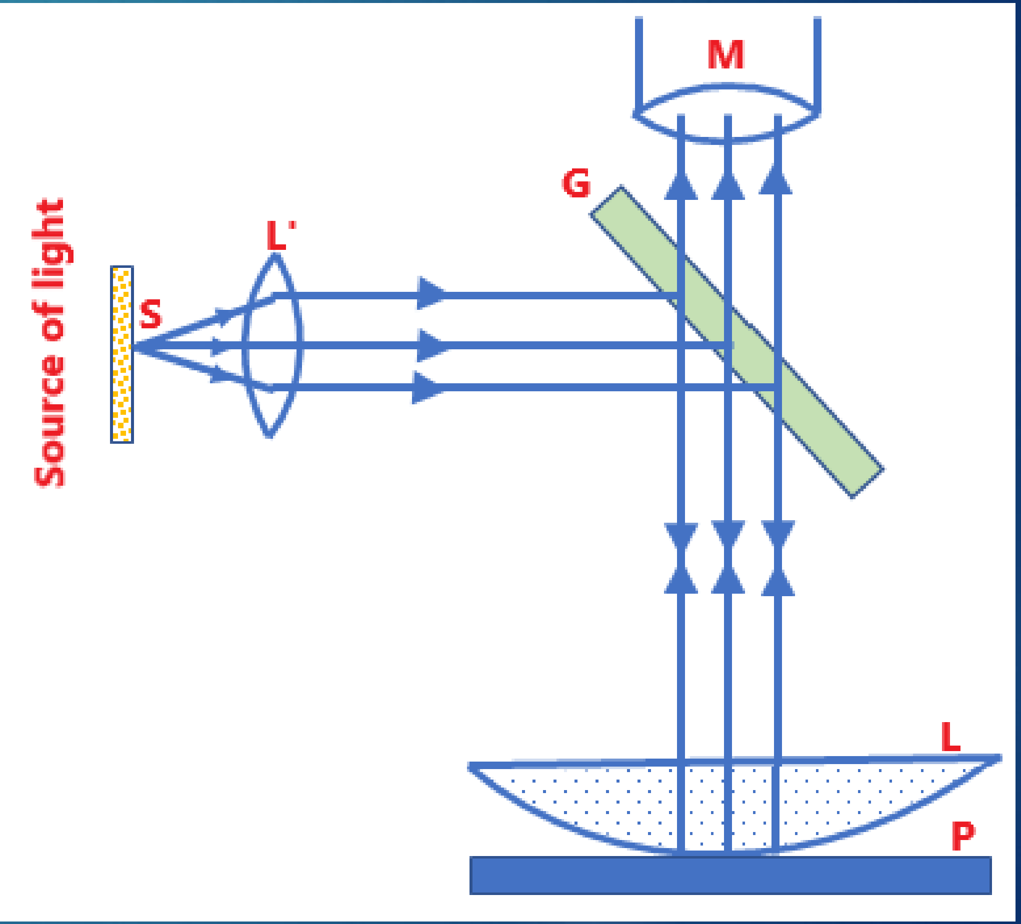
If slab is made up of glass of ref. index 1.5, then ref. index of film should be 1.22. However, in practice transparent material which can be suitably deposited on the glass is MgF_2 having ref. index 1.38.

Colour of non-reflecting film: Non-reflecting films generally appear to be purple in colour when seen in white light, because middle part of the visible spectrum is predominantly absent and resultant colour is purple.

NEWTON'S RINGS

Experimental set up

The apparatus consists of an extended source of light from which a parallel beam of light is obtained which is incident on a glass plate G inclined at 45^0 to the beam. This plate reflects the beam downwards, which in turn is incident normally on the convex lens L placed on a plane glass plate P. The reflection of beam from the lower surface of the lens and upper surface of the plate produces circular interference pattern which may be observed through the microscope M



NEWTON'S RINGS

Mathematical analysis of Newton's Rings

Let the radius of curved surface of the lens be R. Consider a point K on the plate, such that the thickness of the film at that point be 't'. All such points will lie on a circle with O as centre and 'r' as the radius. From Fig. we find that:

$$DA \times AE = OA \times AB = OA(OB - OA)$$

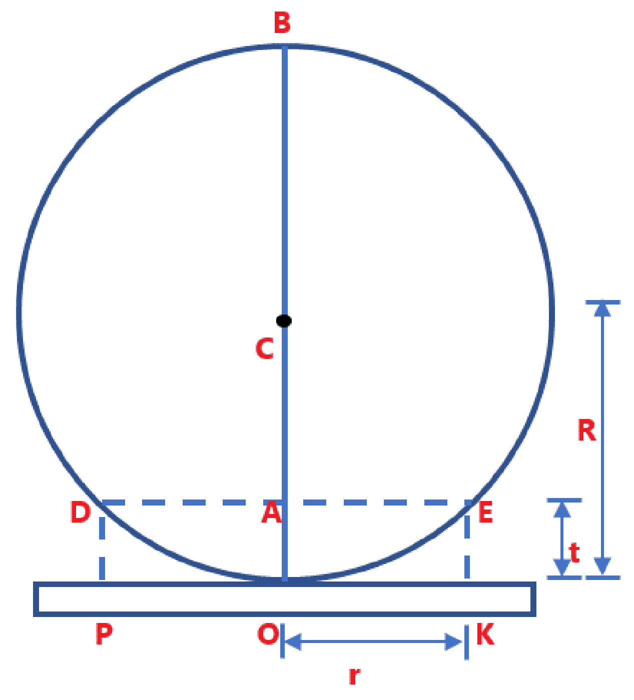
Here DA $= AE = r, OA = t, OB = 2R$, Therefore:

$$r^2 = t(2R - t)$$

If 2R>>t, therefore, t may be neglected as compared to 2R and therefore:

$$r^2 = 2Rt$$

or $$t = \frac{r^2}{2R} \qquad \text{....(58)}$$



NEWTON'S RINGS

Newton's rings by reflected light

If light is incident normally, then for minima or dark ring, we have: $2nt = p\lambda$ ....(59)

And for bright ring: $2nt = (2p+1)\frac{\lambda}{2}$ ....(60)

If r_p is the radius of p^{th} fringe, then the corresponding value of thickness according to (58) will be given by:

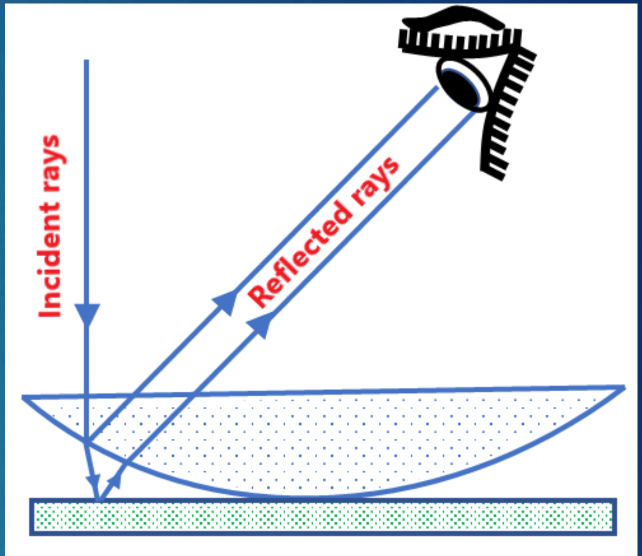
$$t = \frac{r_p^2}{2R}$$

Substituting in (58), and for air n=1, we get:

$$2n\frac{r_p^2}{2R} = p\lambda \quad \text{or} \quad r_p = [p\lambda R]^{\frac{1}{2}} \qquad ....(61)$$

So, the diameter of p^{th} dark ring is given by:

$$D_p = 2r_p = 2[p\lambda R]^{\frac{1}{2}} \qquad .....(62)$$



NEWTON'S RINGS

Similarly for bright fringe substituting in (60) and taking n=1 for air, we get:

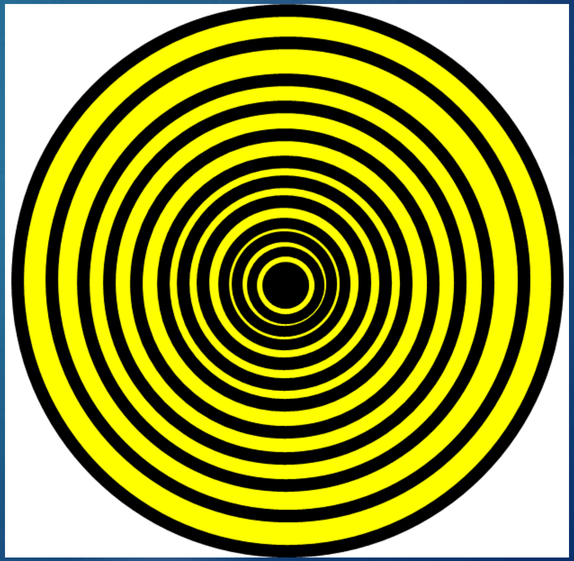
$$2n\frac{r_p^2}{2R} = (2p+1)\frac{\lambda}{2} \qquad \text{or} \qquad r_p = \left[\frac{(2p+1)\lambda R}{2}\right]^{\frac{1}{2}} \qquad \text{....(63)}$$

So, the diameter of p^th bright ring is given by:

$$D_p = 2r_p = 2\left[\frac{(2p+1)\lambda R}{2}\right]^{\frac{1}{2}} \qquad \text{....(64)}$$

Thus we find that diameter of the fringe depends upon its order p, the radius of curvature of the curved surface of the lens and wavelength of the light. Diameter of dark ring is proportional to square root of natural number and that of bright ring is proportional to square root of odd natural number divided by 2.

Fringe pattern obtained is shown in fig.



NEWTON'S RINGS

Newton's rings by transmitted light

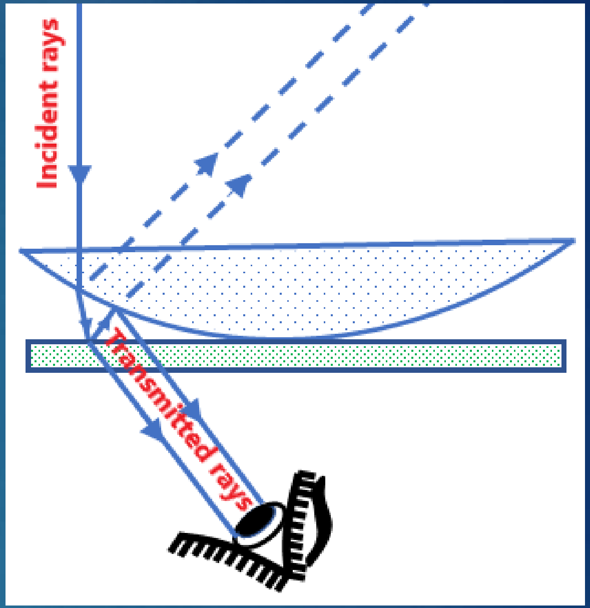
In the transmitted light, the additional path difference of λ/2 due to reflection at the denser medium is not obtained and therefore the expressions for the dark and bright fringes are reversed.

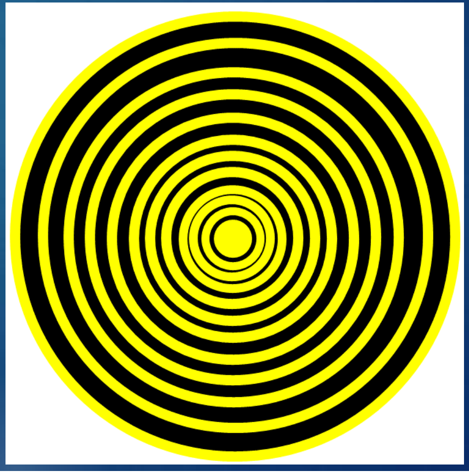
i.e. for dark fringe: $D_p = 2\left[\frac{(2p+1)\lambda R}{2}\right]^{\frac{1}{2}}$ ....(65)

And for bright fringe: $D_p = 2[p\lambda R]^{\frac{1}{2}}$ ....(66)

Fringe pattern obtained is shown in fig.

In the reflected system, central fringe is dark but that in transmitted system is bright. Thus two systems are complimentary to each other. However contrast of the fringes in transmitted system is poor as compared to reflected system.





NEWTON'S RINGS

Applications of Newton's rings

(i) Determination of wavelength of light

If D_p and D_q be the diameters of the p^{th} and q^{th} dark ring, then:

$$D_p = 2[p\lambda R]^{\frac{1}{2}} \quad and \quad D_q = 2[q\lambda R]^{\frac{1}{2}}$$

Therefore: $D_p^2 - D_q^2 = 4(p-q)\lambda R$

or $$\lambda = \frac{D_p^2 - D_q^2}{4(p-q)R} \qquad ....(67)$$

By measuring the diameters of p^{th} and q^{th} ring with microscope and radius of curvature of curved surface with spherometer, we can find wavelength of light.

(ii) Determination of refractive index of liquid

The space between plate and convex lens is filled with the given liquid. The condition for dark ring in this case becomes:

NEWTON'S RINGS

$$2nt = p\lambda$$ Where n is refractive index of liquid.

Using $t = \frac{r_p^2}{2R}$, it becomes:

$$2n\frac{r_p^2}{2R} = p\lambda \qquad \text{or} \qquad n = \frac{p\lambda R}{r_p^2}$$

Knowing p, λ, R and r_p , we can find refractive index.

But generally we find the diameters of p^{th} and q^{th} order dark rings, given by:

$$D_p = 2\left[\frac{p\lambda R}{n}\right]^{\frac{1}{2}} \qquad \text{and} \qquad D_q = 2\left[\frac{q\lambda R}{n}\right]^{\frac{1}{2}}$$

Hence: $D_p^2 - D_q^2 = \frac{4(p-q)\lambda R}{n}$ or $n = \frac{4(p-q)\lambda R}{D_p^2 - D_q^2}$

Newton's rings with white light: Diameter of newton's ring depends upon wavelength of light used. Hence if white light is used, central spot will be white while all other rings will be coloured and only few rings will be observed.