FILTER CIRCUITS

The output of a rectifier is not pure d.c., but it is a pulsating d.c. i.e. it contains a.c. component also. The ratio of a.c. and d.c. components in the output of a rectifier is called ripple factor. Ripple factor should be as small as possible. By using various circuits we can reduce the ripple factor, but ripple factor cannot be made zero. Such circuits which are used to reduce ripple factor i.e. decrease the a.c. component in the output of a rectifier are called filter circuits.

Hence filter circuit is a device which removes (reduces) the a.c. component of rectifier output but allows the d.c. component to reach the load.

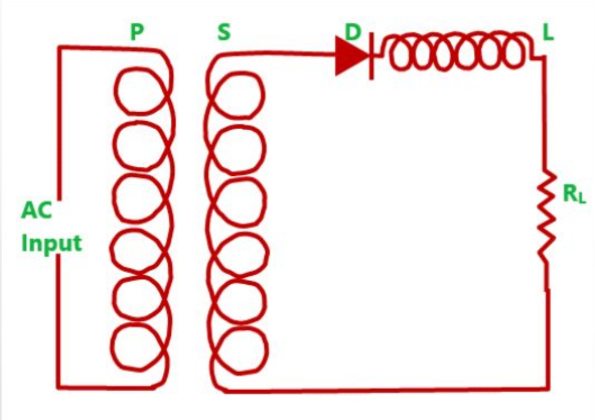
Commonly used filter circuits are:

1. *Series inductor filter or L-filter or LR filter*
2. *Shunt capacitor filter or CR filter*
3. *Choke input filter or LC filter or L-section filter*
4. *Capacitor input filter or π-section filter*

SERIES INDUCTOR FILTER

FOR HALF WAVE RECTIFIER

An inductor is connected in series with the load resistance R_L. We know that inductor opposes any change in the current flowing through it due to self inductance. Hence it opposes a.c. but gives easy path to d.c. The reactance of inductor is given by: $X_L = \omega L$ where L is self inductance of inductor. For d.c. $\omega = 0$, hence $X_L = 0$ i.e. ideal inductor does not offer any reactance to the flow of d.c. Circuit diagram of series inductor filter using half wave rectifier is shown in fig.



Mathematical analysis

Let applied input voltage to diode be given by:

$$E = E_m \sin \omega t \qquad ....(27)$$

If I is current in the circuit, then:

$$E - L\frac{dI}{dt} = IR_L \quad \text{or} \quad E = L\frac{dI}{dt} + IR_L$$

Using (27), we get:

$$L\frac{dI}{dt} + IR_L = E_m \sin \omega t \qquad ,,,,(28)$$

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The solution of equation (28) consists of two parts:

(i) Complimentary function and (ii) particular integral

The complimentary function can be obtained by putting the left hand side of equation (28) equal to zero and solving it for the current I. We have:

$$L\frac{dI}{dt} + IR_L = 0 \quad \text{or} \quad L\frac{dI}{dt} = -IR_L \quad \text{or} \quad \frac{dI}{I} = -\frac{R_L}{L}dt$$

Integrating both sides: $\int \frac{dI}{I} = -\frac{R_L}{L}\int dt$ or $lnI = -\frac{R_L}{L}t + C$

Taking antilog on both sides: $I = Aexp\left(-\frac{R_L}{L}t\right)$ Where A is some constant. ....(29)

The particular integral can be obtained by solving equation (28). We have:

$$I = \frac{E_m}{\sqrt{R_L^2 + \omega^2 L^2}}\sin(\omega t - \emptyset) \qquad ....(30)$$

Where $\emptyset = \tan^{-1}\frac{\omega L}{R_L}$

Thus diode current is given by: $I = Aexp\left(-\frac{R_L}{L}t\right) + \frac{E_m}{\sqrt{R_L^2 + \omega^2 L^2}}\sin(\omega t - \emptyset)$ ....(31)

Since at t = 0, I = 0, equation (31) becomes:

SERIES INDUCTOR FILTER

$$0 = A + \frac{E_m}{\sqrt{R_L^2+\omega^2 L^2}} \sin(-\emptyset) = A - \frac{E_m}{\sqrt{R_L^2+\omega^2 L^2}} \sin \emptyset$$

or $$A = \frac{E_m}{\sqrt{R_L^2+\omega^2 L^2}} \sin \emptyset \qquad \text{....(32)}$$

Substituting in (31), we get:

$$I = \frac{E_m}{\sqrt{R_L^2+\omega^2 L^2}} \sin \emptyset \, exp\left(-\frac{R_L}{L} t\right) + \frac{E_m}{\sqrt{R_L^2+\omega^2 L^2}} \sin(\omega t - \emptyset)$$

or $$I = \frac{E_m}{\sqrt{R_L^2+\omega^2 L^2}} \left[exp\left(-\frac{R_L}{L} t\right) \sin \emptyset + \sin(\omega t - \emptyset)\right] \qquad \text{....(33)}$$

Second term in above equation is sinusoidal and becomes zero for certain values of ωt. The first exponential term is always positive and decreases with time. If L/R_L is large the first term will decrease very slowly and I becomes zero for certain value of angle say $\theta_2 = \omega t$ called cut off angle. To find cut off angle use the condition that at $\omega t = \theta_2$, I = 0. So equation (33) becomes:

$$0 = \frac{E_m}{\sqrt{R_L^2+\omega^2 L^2}} \left[exp\left(-\frac{\theta_2 R_L}{\omega L}\right) \sin \emptyset + \sin(\theta_2 - \emptyset)\right]$$

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or $\sin(\theta_2 - \emptyset) + exp\left(-\frac{\theta_2 R_L}{\omega L}\right)\sin\emptyset = 0$

or $\sin\theta_2\cos\emptyset - \cos\theta_2\sin\emptyset + exp\left(-\frac{\theta_2 R_L}{\omega L}\right)\sin\emptyset = 0$

or $\sin\theta_2\cos\emptyset = \sin\emptyset\left[\cos\theta_2 - exp\left(-\frac{\theta_2 R_L}{\omega L}\right)\right]$

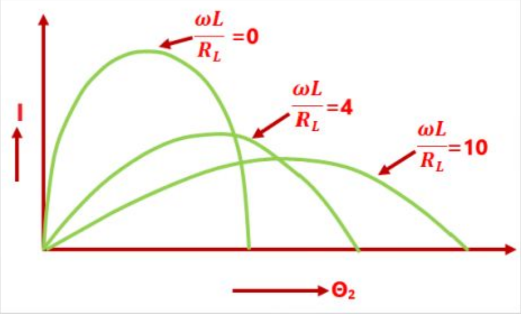
or $\frac{\cos\emptyset}{\sin\emptyset} = \frac{\cos\theta_2 - exp\left(-\frac{\theta_2 R_L}{\omega L}\right)}{\sin\theta_2}$ or $\cot\emptyset = \frac{\cos\theta_2 - exp\left(-\frac{\theta_2 R_L}{\omega L}\right)}{\sin\theta_2}$ ....(34)

Since $\tan\emptyset = \frac{\omega L}{R_L}$ or $\cot\emptyset = \frac{R_L}{\omega L}$

Therefore equation (34) becomes:

$$\frac{R_L}{\omega L} = \frac{\cos\theta_2 - exp\left(-\frac{\theta_2 R_L}{\omega L}\right)}{\sin\theta_2}$$ .....(35)

If $\frac{R_L}{\omega L}$ is known θ_2 can be determined from equation (35). Graphs between value of current obtained and θ_2 for various values of $\frac{\omega L}{R_L}$ is shown in fig.



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Ripple factor

According to Fourier analysis, the output of half wave rectifier is given by:

$$\mathrm{E} = \frac{E_m}{\pi} + \frac{E_m}{2} sin\omega t - \frac{2}{3}\frac{E_m}{\pi} cos2\omega t - \frac{2}{15}\frac{E_m}{\pi} cos4\omega t + \cdots$$

First term represents d.c. component: $E_{dc} = \frac{E_m}{\pi}$ **or** $I_{dc} = \frac{E_{dc}}{R_L} = \frac{E_m}{\pi R_L}$ **.....(36)**

Impedance of the inductor and load resistance for first component is given by:

$$Z_1 = \sqrt{R_L^2 + \omega^2 L^2}$$

And impedance of the inductor and load resistance for the second component is given by:

$$Z_2 = \sqrt{R_L^2 + (2\omega L)^2} = \sqrt{R_L^2 + 4\omega^2 L^2}$$

As $R_L^2 \ll \omega L$, **Hence** $Z_1 \cong \omega L$ **and** $Z_2 \cong 2\omega L$ **....(37)**

Peak value of voltage due to first a.c. component, $E_{01} = \frac{E_m}{2}$

R.M.S. value of voltage due to first a.c. component, $E_1 = \frac{E_{01}}{\sqrt{2}} = \frac{E_m}{2\sqrt{2}}$

R.M.S. value of current due to first component, $I_1 = \frac{E_1}{Z_1} = \frac{E_m}{2\sqrt{2}}\frac{1}{\sqrt{R_L^2+\omega^2 L^2}}$ **....(38)**

Similarly:

SERIES INDUCTOR FILTER

Peak value of voltage due to second a.c. component, $E_{02} = \frac{2E_m}{3\pi}$

R.M.S. value of voltage due to second a.c. component, $E_2 = \frac{E_{02}}{\sqrt{2}} = \frac{2E_m}{3\pi\sqrt{2}}$

R.M.S. value of current due to second component, $I_2 = \frac{E_2}{Z_2} = \frac{2E_m}{3\pi\sqrt{2}} \frac{1}{\sqrt{R_L^2 + 4\omega^2 L^2}}$ ....(39)

Now dividing (38) and (39), we get:

$$\frac{I_1}{I_2} = \frac{\frac{E_m}{2\sqrt{2}} \frac{1}{\sqrt{R_L^2+\omega^2 L^2}}}{\frac{2E_m}{3\pi\sqrt{2}} \frac{1}{\sqrt{R_L^2+4\omega^2 L^2}}}$$

Neglecting R_L, we get:

$$\frac{I_1}{I_2} = \frac{\frac{E_m}{2\sqrt{2}} \frac{1}{\sqrt{\omega^2 L^2}}}{\frac{2E_m}{3\pi\sqrt{2}} \frac{1}{\sqrt{4\omega^2 L^2}}} = \frac{E_m}{2\sqrt{2}\omega L} . \frac{3\pi\sqrt{2} \times 2\omega L}{2E_m} = \frac{3\pi}{2} = 4.7$$

i.e. current due to first component of a.c. is about 5 times the current due to second component. Hence ripple factor is mainly due to first component and higher components can be neglected.

SERIES INDUCTOR FILTER

Now ripple factor is given by:

$$\gamma = \frac{I_{ac}}{I_{dc}} = \frac{\frac{E_m}{2\sqrt{2}} \frac{1}{\sqrt{R_L^2+\omega^2 L^2}}}{\frac{E_m}{\pi R_L}} = \frac{\pi R_L}{2\sqrt{2}\sqrt{R_L^2+\omega^2 L^2}} = 1.11 \frac{R_L}{\sqrt{R_L^2+\omega^2 L^2}}$$ **Using (36) and (38)**

If $R_L << \omega L$, then R_L can be neglected.

$$\gamma = 1.11 \frac{R_L}{\omega L}$$ **....(40)**

Hence ripple factor will decrease if load resistance is small.

If load resistance is large i.e. $R_L >> \omega L$, then ωL can be neglected. Then

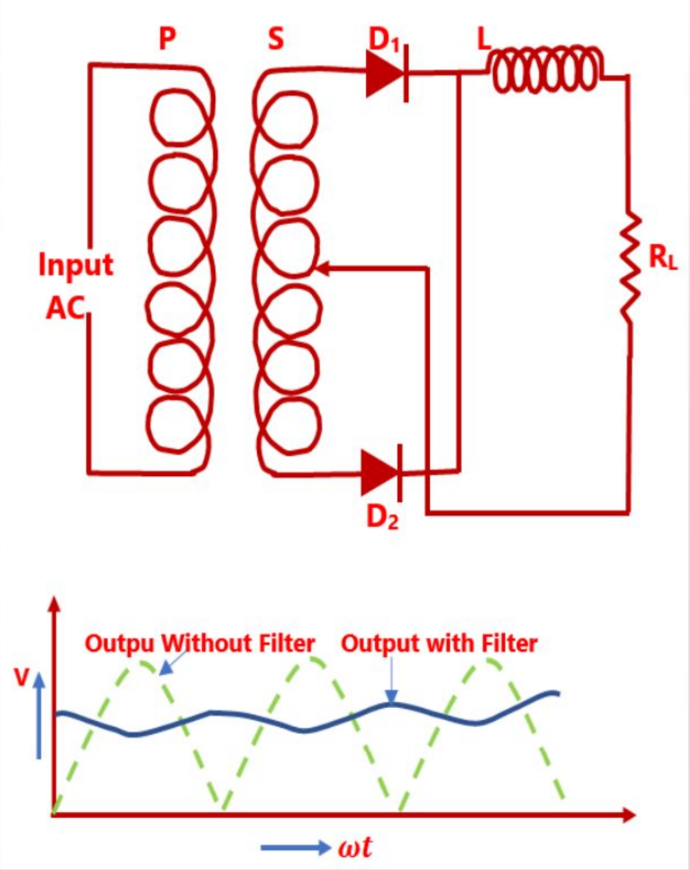
$$\gamma = 1.11$$

Which is almost same as the ripple factor of half wave rectifier without series inductor filter. Hence, we conclude that series inductor filter for half wave rectifier is suitable only if load is small.

SERIES INDUCTOR FILTER

FOR FULL WAVE RECTIFIER

Circuit diagram of series inductor filter using full wave rectifier is shown in fig. Working is same as explained for half wave rectifier. The output wave form without filter and with filter is also shown in fig.



Ripple factor

According to Fourier analysis, the output of full wave rectifier is given by:

$$\mathrm{E} = \frac{2E_m}{\pi} - \frac{4}{3}\frac{E_m}{\pi} cos2\omega t - \frac{4}{15}\frac{E_m}{\pi} cos4\omega t + \cdots$$

First term represents d.c. component:

$$E_{dc} = \frac{2E_m}{\pi} \quad \text{or} \quad I_{dc} = \frac{E_{dc}}{R_L} = \frac{2E_m}{\pi R_L} \qquad .....(41)$$

Impedance of the inductor and load resistance for second harmonic component is given by: $Z_2 = \sqrt{R_L^2 + (2\omega L)^2} = \sqrt{R_L^2 + 4\omega^2 L^2}$

Impedance of the inductor and load resistance for the fourth harmonic component is given by: $Z_4 = \sqrt{R_L^2 + (4\omega L)^2} = \sqrt{R_L^2 + 16\omega^2 L^2}$

SERIES INDUCTOR FILTER

Peak value of voltage due to second harmonic a.c. component, $E_{02} = \frac{4E_m}{3\pi}$

R.M.S. value of voltage due to second harmonic a.c. component, $E_2 = \frac{E_{02}}{\sqrt{2}} = \frac{4E_m}{3\pi\sqrt{2}}$

R.M.S. value of current due to second harmonic component, $I_2 = \frac{E_2}{Z_2} = \frac{4E_m}{3\pi\sqrt{2}} \frac{1}{\sqrt{R_L^2+4\omega^2 L^2}}$ ....(42)

Peak value of voltage due to fourth harmonic a.c. component, $E_{04} = \frac{4E_m}{15\pi}$

R.M.S. value of voltage due to fourth harmonic a.c. component, $E_4 = \frac{E_{04}}{\sqrt{2}} = \frac{4E_m}{15\pi\sqrt{2}}$

R.M.S. value of current due to fourth harmonic component, $I_4 = \frac{E_4}{Z_4} = \frac{4E_m}{15\pi\sqrt{2}} \frac{1}{\sqrt{R_L^2+16\omega^2 L^2}}$ ....(43)

Now dividing (42) and (43), we get:

$$\frac{I_2}{I_4} = \frac{\frac{4E_m}{3\pi\sqrt{2}} \frac{1}{\sqrt{R_L^2+4\omega^2 L^2}}}{\frac{4E_m}{15\pi\sqrt{2}} \frac{1}{\sqrt{R_L^2+16\omega^2 L^2}}}$$

Neglecting R_L, we get:

SERIES INDUCTOR FILTER

$$\frac{I_2}{I_4} = \frac{\frac{4E_m}{3\pi\sqrt{2}}\frac{1}{\sqrt{4\omega^2L^2}}}{\frac{4E_m}{15\pi\sqrt{2}}\frac{1}{\sqrt{16\omega^2L^2}}} = \frac{4E_m}{3\pi\sqrt{2}\times 2\omega L}\cdot\frac{15\pi\sqrt{2}\times 4\omega L}{4E_m} = 10$$

i.e. current due to second harmonic component of a.c. is about 10 times the current due to fourth harmonic component. Hence ripple factor is mainly due to second harmonic component and higher components can be neglected.

Now ripple factor is given by:

$$\gamma = \frac{I_{ac}}{I_{dc}} = \frac{\frac{E_m}{3\pi\sqrt{2}}\frac{1}{\sqrt{R_L^2+4\omega^2L^2}}}{\frac{2E_m}{\pi R_L}} = \frac{2R_L}{3\sqrt{2}R_L\sqrt{1+\frac{4\omega^2L^2}{R_L^2}}} = 0.47\frac{1}{\sqrt{1+\frac{4\omega^2L^2}{R_L^2}}}$$ Using (41) and (42)

If R_L << ωL, then R_L can be neglected.

$$\gamma = 0.47\frac{R_L}{2\omega L} = 0.235\frac{R_L}{\omega L}$$ ....(44)

Hence ripple factor will decrease if load resistance is small.

If load resistance is large i.e. R_L >> ωL, then ωL can be neglected. Then $\gamma = 0.47$

Which is almost same as the ripple factor of full wave rectifier without series inductor filter. Hence, we conclude that series inductor filter for full wave rectifier also is suitable only if load is small.