MICHELSON’S INTERFEROMETER

The interferometer is a device to exploit the interference of light for some scientific purpose. The Michelson Interferometer was first devised by Albert Michelson in 1881, and was used to detect the motion of earth through ether or to detect the so called hypothetical ether as the medium through which light waves were supposed to propagate.

Experimental set up

it consists of two well polished mirrors M_1 and M_2 placed perpendicular to each other. GP is a half-silvered glass plate equally inclined to the mirrors M_1 and M_2. It is called beam splitter. When parallel beam of light is incident on plate GP, a part of the light is reflected and remaining part of almost same amplitude is transmitted. The refracted beam (1) goes to the mirror M_1. From the mirror, the incident beam is reflected along the same path and is then incident on the plate GP, from where it is reflected downwards. The reflected beam (2) goes to the mirror M_2 and is incident normally on it. From the mirror M_2 , the incident beam is reflected along the same path and is then transmitted through GP and joints the beam reflected from mirror M_1.

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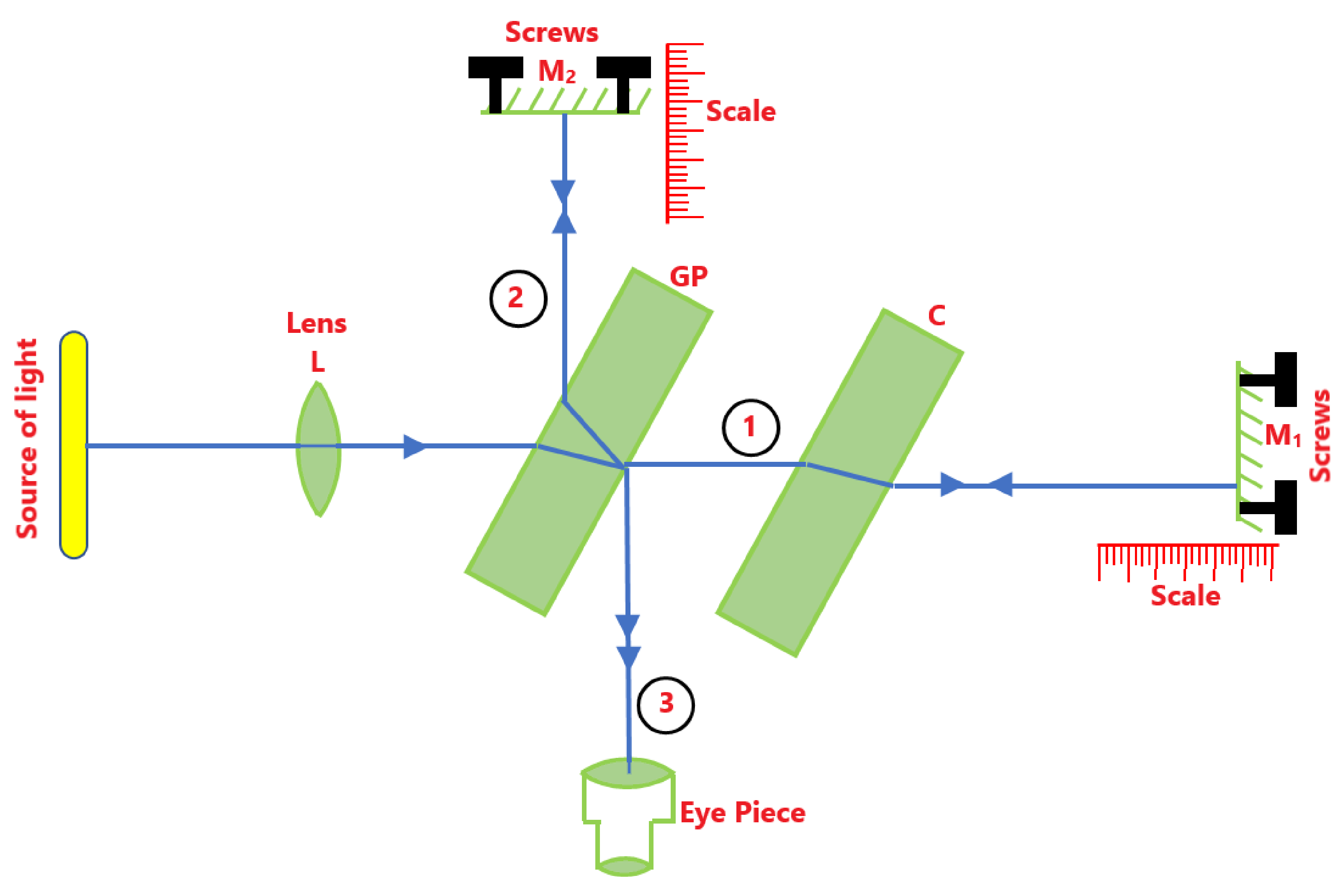

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Thus the beam (3) contains light reflected from both M_1 and M_2. The reflected beam (2) passes through plate GP thrice, but the beam (1) passes through it only once. So, to compensate for it, a plate C of same material and same thickness as that of GP is placed parallel to GP in the path of beam (1). The optical paths of two parts of beam (3) can be varied and they interfere to produce interference fringes, which can be seen through an eye piece.

Formation of fringes

The shape of the fringes is determined by the mutual inclination of the mirrors M_1 and M_2. To simplify the analysis, we consider an equivalent optical system having a single optical axis, perpendicular to the mirror M_2. Mirror M_1 is replaced by its image M_1' in the mirror M_2. Also the source S is replaced by its image S' in the mirror M_2. P is any point on the source image S' and a ray of light inclined to the optical axis at an angle θ is incident on M_1' and M_2.

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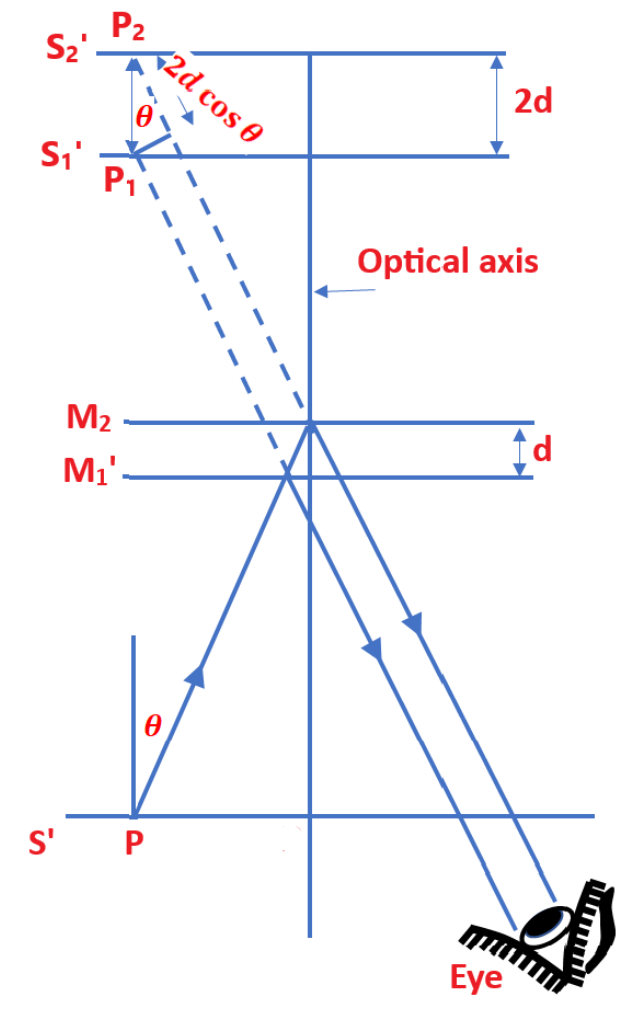
The reflected beam appears to come from the virtual images P_1 and P_2 of the source S' in the M_1' and M_2. If the separation between the mirrors M_1' and M_2 is 'd' then the separation between S_1' and S_2' will be '2d'. Hence the optical path difference between the reflected beams going to eye piece will be:

$$\Delta = 2d\cos\theta$$

Beam (1) experience a reflection from denser medium from plate GP. But the reflection of beam (2) from GP is at rarer medium. So, an additional path difference of λ/2 is introduce in the beam reflected from mirror M_1. Hence, the optical path difference between the interfering beams will be:

$$\Delta = 2d\cos\theta + \frac{\lambda}{2} \qquad \text{.....(68)}$$



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Hence condition for minima is: $\Delta = (2p+1)\frac{\lambda}{2}$

Hence for minima: $2d\cos\theta + \frac{\lambda}{2} = (2p+1)\frac{\lambda}{2}$ or $2d\cos\theta = p\lambda$

For maxima: $2d\cos\theta + \frac{\lambda}{2} = p\lambda$ or $2d\cos\theta = (2p-1)\frac{\lambda}{2}$

Circular fringes-fringes of equal inclination

The interference produced by the equivalent optical system is a case of interference by an air film of same thickness, illuminated by an extended source of light. Hence the fringes of equal inclination are produced. so, the interference pattern consists of circular fringes with their centre on the optical axis. The fringes are localized at infinity and can be seen by the eye or a telescope focused at infinity.

Case 1: If d=0. That is geometrical distance of the two mirrors from the plate GP is same, then:

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$$\Delta = 2d\cos\theta + \frac{\lambda}{2} = \frac{\lambda}{2}$$

Hence central fringe will be dark.

Case 2: If we move one of the mirror parallel to itself, d starts increasing and so path difference also increases. For the rays of light incident along the optic axis of the system $\theta=0$ and hence:

$$\Delta = 2d\cos 0 + \frac{\lambda}{2} = \quad or \quad \Delta = 2d + \frac{\lambda}{2}$$

When d=λ/4, we have: $\Delta = 2\times\frac{\lambda}{4} + \frac{\lambda}{2} = \lambda$

So, the centre of interference pattern becomes a maximum. Again moving the mirror further by λ/4 we get minimum at the centre. Thus, by moving one of the mirrors parallel to itself, the central fringe becomes a maximum and minimum alternatively.

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Case 3: If we look on the fringe pattern obliquely at an angle θ, we observe the circular fringe, say a minimum for which when d increases, 2d cos θ increases.

$$2d\cos\theta = p\lambda$$

But for a fringe of p^{th} order it should remain constant. i.e.

$$2d\cos\theta = constant \qquad \text{or} \qquad \cos\theta \propto \frac{1}{d}$$

So, when d increases, for the fringe of p^{th} order cos θ decreases or θ increases. In other words, as one of the mirrors is moved parallel to itself, the fringe pattern appear to be expanding outwards and the central fringe becomes a maximum and minimum alternatively.

Localized fringes

If the mirrors are perpendicular to each other, the air film trapped between the mirrors will have same thickness every where and we get circular fringes. However, if mirrors are inclined to each other at some angle, the air film between M_1 and M_2 becomes wedge shaped and we get localized fringes of

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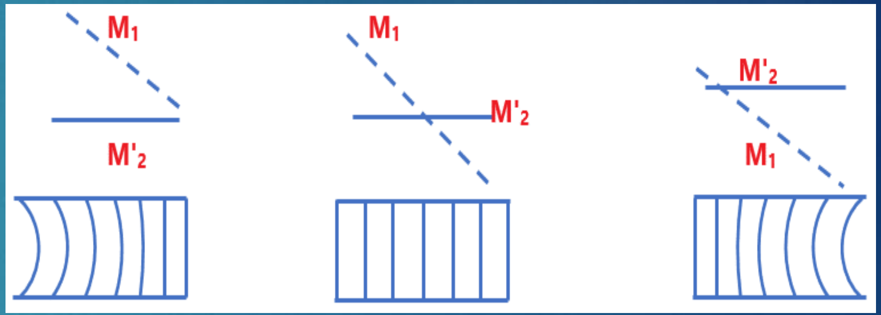
equal thickness located between S_1' and S_2'. When one of the mirrors is moved these fringes move across the field of view.

The shape of the fringes observed for various values of inclination between the mirrors are shown in fig.

The fringes are curved and are always convex towards the thin edge of wedge. Fringes are straight as shown in fig. (b), when M_2 actually intersects M_1' in the middle.



Order of central fringe

For the central fringe, $\theta = 0$, and so if it is dark then:

$$2d = p\lambda \text{ or } \qquad p = \frac{2d}{\lambda}$$

The above equation shows that the order of central fringe is not zero, unless d=0. on the other hand for the given interference pattern, the order of central fringe is maximum. As θ increases, cos θ decreases, so order p decreases.

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APPLICATIONS OF MICHELSON'S INTERFEROMETER

(i) Determination of wavelength of light

For central dark fringe, we have $\theta = 0$, Therefore:

$$2d\cos\theta = p\lambda \quad \text{gives} \quad 2d = p\lambda \quad \text{or} \quad \lambda = \frac{2d}{p}$$

So, the two mirrors are adjusted exactly perpendicular to each other and circular fringes are obtained. One of the mirrors is moved parallel to itself and the number (m) of fringes crossing the field of view is counted. Suppose the distance between M_1' and M_2 for the p^{th} order central dark fringe be d_1 and that for $(p+m)^{th}$ order central dark fringe be d_2, then:

$$2d_1 = p\lambda \quad \text{and} \quad 2d_2 = (p+m)\lambda$$

Subtracting: $2(d_2 - d_1) = m\lambda$

or $\lambda = \frac{2(d_2-d_1)}{m} = \frac{2\Delta d}{m}$ So, we can find wavelength.

(ii) Measurement of small difference in wavelength

Suppose source of light emits two wavelengths λ_1 and λ_2 such that $\lambda_1 \approx \lambda_2$. i.e. difference in wavelength is small. Two interference patterns are formed, which are superimposed upon each other. Let at certain point maxima of two patterns are superimposed. Intensity at that point will be maximum. Now we change the distance between mirrors by moving one of the mirrors parallel to itself. Two patterns will get out of step and intensity at observation point will decrease. If we further go on moving the mirror, the intensity will again increase and attains the initial value. This change from one state to other will occur when the number (m_1) of fringes of one wavelength say (λ_1) crossing the field of view is 1 more than that (m_2) for (λ_2). Now if change in separation of M_1' and M_2 be Δd, Then:

$$m_1 = \frac{2\Delta d}{\lambda_1} \quad \text{and} \quad m_2 = \frac{2\Delta d}{\lambda_2}$$

But $\quad m_1 - m_2 = 1$

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Therefore: $\frac{2\Delta d}{\lambda_1} - \frac{2\Delta d}{\lambda_2} = 1$

or $\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{2\Delta d}$ or $\frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2} = \frac{1}{2\Delta d}$

Since $\lambda_2 \approx \lambda_1$ so $\lambda_1 \lambda_2 = \lambda^2$ (Where λ is mean wavelength)

or $\lambda_2 - \lambda_1 = \frac{\lambda^2}{2\Delta d}$ or $\Delta\lambda = \frac{\lambda^2}{2\Delta d}$

(iii) Determination of refractive index of material or thickness of thin sheet

A thin sheet of thickness 't' whose refractive index 'n' is to be measured is placed in the path of light rays going towards mirror M_1, then:

Optical path of sheet= $nt - t = (n-1)t$

Increase in optical path towards mirror M_1= $(n-1)t$

Increase in optical path after reflection from mirror M_1= $(n-1)t$

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So, total increase in optical path= $2(n-1)t$

Net path difference produced by transparent sheet= $2(n-1)t$

If (m) fringes cross the field of view, then:

$$2(n-1)t = m\lambda$$

or $$n = \frac{m\lambda}{2t} + 1$$

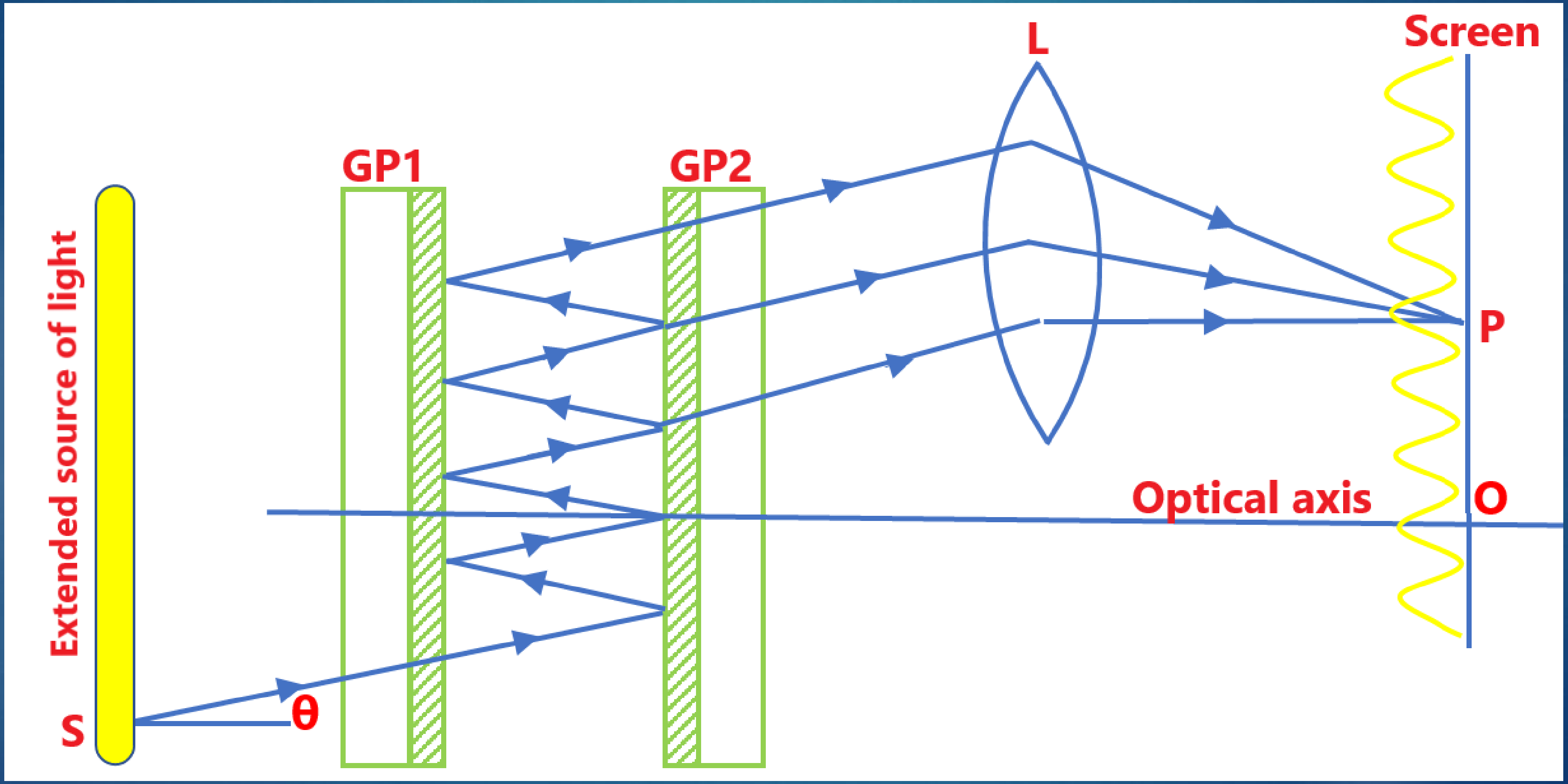
From this relation, we can find refractive index of material of sheet, if its thickness is known.

Or we can find thickness of sheet if its refractive index is known by using the relation:

$$t = \frac{m\lambda}{2(n-1)}$$

Fabry Perot Interferometer and Etalon

Principle: It is based on the principle of interference by multiple reflections.



Fabry Perot Interferometer and Etalon

Construction: It consists of two thick glass or quartz plates, placed parallel to each other, so that a film of air is enclosed between them. This air film forms the medium in which the multiple reflections occur. The inner surface of the plates are coated with thin layer of silver or aluminium (about 50nm) to make it

partially reflecting. The outer surfaces of the plates are made very very slightly inclined w.r.t. the inner surfaces, to avoid the spurious interference due to the plates themselves acting as the medium for multiple reflections.

Working: The transmitted light is brought to focus on the screen with the help of a lens. If the distance between the inner surfaces of plates be (d), then the point P at which the light rays are focused, will be a maxima if:

$$2d\cos\theta = p\lambda$$

Where p=0,1,2,3,……..

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The fringe pattern consists of alternate bright and dark circular rings with centre on the optical axis (O). These are fringes of equal inclination called Haidinger fringes.

In actual interferometer, one plate is kept fixed, while other plate is capable of moving slowly with the help of slow motion screw. So, the thickness of air film between the plates can be changed. The interference pattern obtained in Fabry Perot interferometer is extremely sharp as compared to Michelson's interferometer. Hence it is very useful for resolving very small wavelength differences.

Etalon: In the etalon two semi-silvered plates are mounted in a framework. Their coated faces are kept in constant parallel position a fixed distance apart by a fixed spacer, which is generally a hollow cylinder of invar or silica with three

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Projecting stud at each end. The plates are kept in place slightly inclined by using springs attached in proper way. Etalons with spacing of length ranging from 1 to 200 mm are used in investigation of hyperfine structure of spectral lines.

Distribution of intensity in Fabry Perot fringes

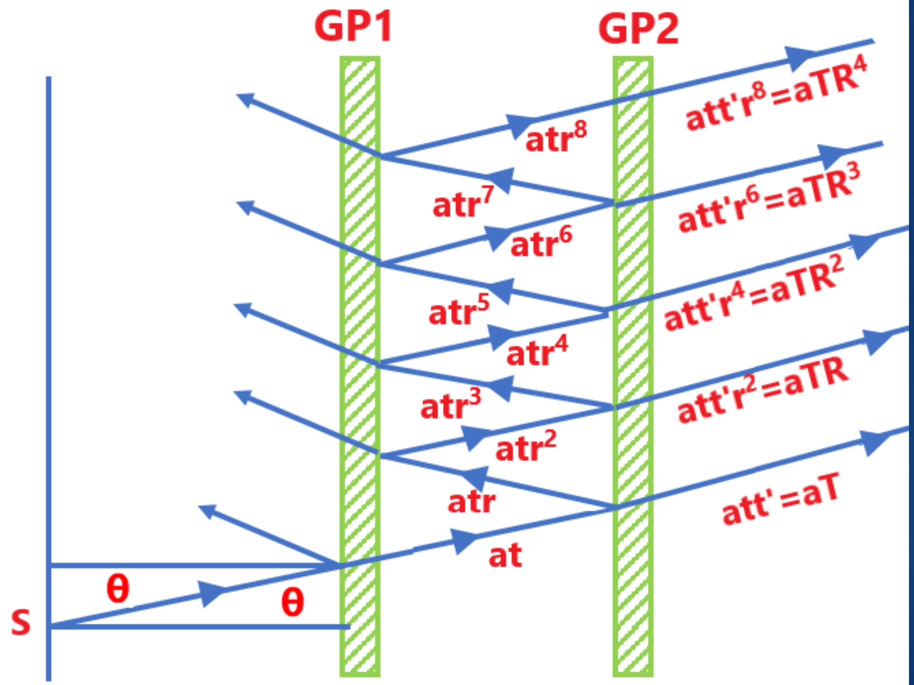
Let amplitude of incident wave = a

Reflection coefficient of surface = R

Transmission coefficient of medium = T

Transmission coefficient of incident wave in passing from air to glass = t

Transmission coefficient of wave in passing from glass to air = t'



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Amplitudes of reflected beams after 1st , 2nd ,3rd , …. reflections will be atr, $atr^2 = atR$, atr^3 , $atr^4 = atR^2$…… respectively.

Amplitudes of transmitted waves after 1st , 2nd ,3rd , …. reflections will be att'=aT, $att'r^2 = aTR$, $att'r^4 = aTR^2$…… respectively.

Common phase difference between successive waves is:

$$\delta = \frac{2\pi}{\lambda} \times path\ difference = \frac{2\pi}{\lambda}(2d\cos\theta) = \frac{4\pi d\cos\theta}{\lambda}$$

The transmitted waves re represented by: $aTe^{i\omega t}, aTRe^{i(\omega t-\delta)}, aTR^2e^{i(\omega t-2\delta)}, \dots \dots$.

Where ω is angular frequency of incident beam.

The resultant amplitude of all the interfering beams is given by:

$$y = aTe^{i\omega t} + aTRe^{i(\omega t-\delta)} + aTR^2e^{i(\omega t-2\delta)} + \cdots$$

or $$y = aTe^{i\omega t}(1 + Re^{-i\delta} + R^2e^{-i2\delta} + \cdots)$$

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But $1 + Re^{-i\delta} + R^2e^{-i2\delta} + \cdots = \frac{1}{1-Re^{-i\delta}}, called\ amplitude\ factor.$

Therefore $y = \frac{aTe^{i\omega t}}{1-Re^{-i\delta}}$ and its complex conjugate $y* = \frac{aTe^{-i\omega t}}{1-Re^{i\delta}}$

Thus resultant intensity: $I = yy* = \frac{aTe^{i\omega t}}{1-Re^{-i\delta}} \cdot \frac{aTe^{-i\omega t}}{1-Re^{i\delta}}$

$$= \frac{a^2T^2}{1-Re^{i\delta}-Re^{-i\delta}+R^2} = \frac{a^2T^2}{1-R(e^{i\delta}+e^{-i\delta})+R^2}$$

$$= \frac{a^2T^2}{1-R(\cos\delta+i\sin\delta+\cos\delta-i\sin\delta)+R^2} = \frac{a^2T^2}{1-2R\cos\delta+R^2}$$

$$= \frac{a^2T^2}{1+R^2-2R(1-2\sin^2\frac{\delta}{2})} = \frac{a^2T^2}{(1-R)^2+4R\sin^2\frac{\delta}{2}}$$

$$\mathrm{I} = \frac{a^2T^2}{(1-R)^2\left[1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}\right]} \qquad \ldots\ldots(69)$$

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Intensity will be maximum when $\sin^2\frac{\delta}{2} = 0$

Therefore maximum intensity, $I_{max} = \frac{a^2T^2}{(1-R)^2}$ .....(70)

Intensity will be minimum when $\sin^2\frac{\delta}{2} = 1$

Therefore minimum intensity, $I_{min} = \frac{a^2T^2}{(1-R)^2\left[1+\frac{4R}{(1-R)^2}\right]} = \frac{a^2T^2}{(1-R)^2+4R} = \frac{a^2T^2}{(1+R)^2}$ .....(71)

Therefore: $\frac{I_{max}}{I_{min}} = \left(\frac{1+R}{1-R}\right)^2$ .....(72)

Using (70), equation (69) can be written as:

$$\mathrm{I} = \frac{I_{max}}{\left[1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}\right]} = \frac{I_{max}}{\left[1+F\sin^2\frac{\delta}{2}\right]} \qquad .....(73)$$

Where $F = \frac{4R}{(1-R)^2}$

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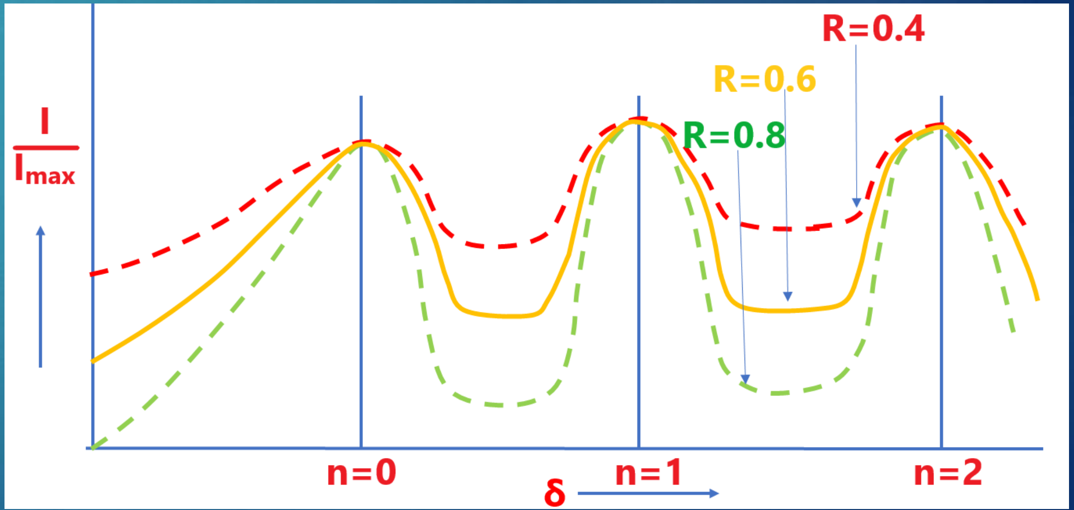
From (73), $$\frac{I}{I_{max}} = \frac{1}{F\sin^2\frac{\delta}{2}} \quad ....(74)$$

This equation shows that $\frac{I}{I_{max}}$ is controlled by (i) F which depends upon R only and (ii) δ which depends upon separation between plates 'd' and the direction of incidence θ.

Graphs between $\frac{I}{I_{max}}$ and δ for various values of R=0.4, 0.6, 0.8 are shown in fig.

From the graph, we find that fall of $\frac{I}{I_{max}}$ is most sharp for R=0.8 and diminishes as R decreases. Thus the visibility and hence the resolution depends upon the rate of change or ratio $\frac{I}{I_{max}}$ with R



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e.g. if R = 0.8, then: $F = \frac{4R}{(1-R)^2} = 80$

Hence : $\mathrm{I} = \frac{I_{max}}{\left[1+F\sin^2\frac{\delta}{2}\right]} = \frac{I_{max}}{\left[1+80\sin^2\frac{\delta}{2}\right]}$

If $\sin^2\frac{\delta}{2} = \frac{1}{80}$

Then: $\mathrm{I} = \frac{I_{max}}{\left[1+80\times\frac{1}{80}\right]} = \frac{I_{max}}{2}$ i.e. intensity falls to one half of the maximum.

Now $\sin^2\frac{\delta}{2} = \frac{1}{80}$ or $\sin\frac{\delta}{2} = (80)^{-1/2}$ or $\delta = 2\sin^{-1}(80)^{-1/2} = 2(n\pi \pm 0.112)$

or $\delta = 2n\pi \pm 0.224$

And for minima, we can write: $I_{min} = \frac{I_{max}}{1+\frac{4R}{(1-R)^2}} = \frac{I_{max}}{81}$ for R = 0.8

Hence a sharp change occurs from maxima to minima for greater value of R.

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e.g. if R = 0.6, then: $F = \frac{4R}{(1-R)^2} = 15$

Hence : $\mathrm{I} = \frac{I_{max}}{\left[1+F\sin^2\frac{\delta}{2}\right]} = \frac{I_{max}}{\left[1+15\sin^2\frac{\delta}{2}\right]}$

If $\sin^2\frac{\delta}{2} = \frac{1}{80}$

Then: $\mathrm{I} = \frac{I_{max}}{\left[1+15\times\frac{1}{80}\right]} = \frac{I_{max}}{1.19}$ i.e. intensity falls to 1/1.19=0.84 of the maximum.

Now $\sin^2\frac{\delta}{2} = \frac{1}{80}$ or $\sin\frac{\delta}{2} = (80)^{-1/2}$ or $\delta = 2\sin^{-1}(80)^{-1/2} = 2(n\pi \pm 0.112)$

or $\delta = 2n\pi \pm 0.224$

And for minima, we can write: $I_{min} = \frac{I_{max}}{1+15} = \frac{I_{max}}{16}$ for R = 0.6

Hence a sharp change will not occurs from maxima to minima for greater value of R.

THANK YOU