GENERAL PROPERTIES OF NUCLEI

MATTER → is that which possesses mass and occupies space

↓

↓ Solid	↓ Liquid	↓ Gas

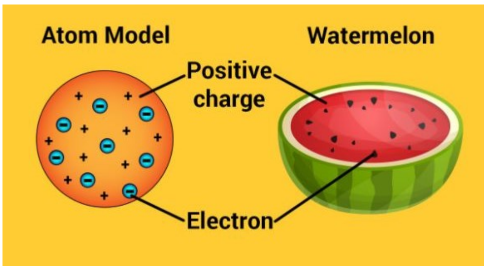
↓

Made up of atoms

Atom: initially : indivisible

Later researches : divisible

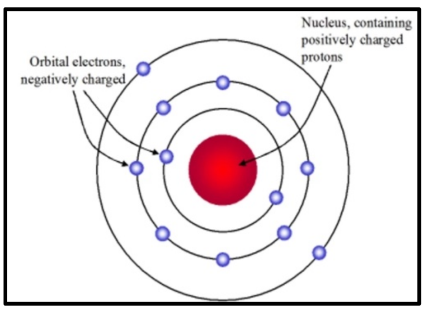
J.J.Thomson gave Plum Pudding model or watermelon model also called static model of atom in 1904.

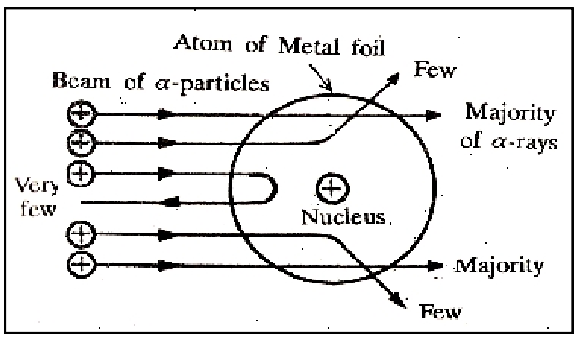


→ atom consists of +ve electric fluid uniformally distributed over the entire body of atom and –ve electrons are embedded inside this fluid like seeds in a watermelon.

→ failed to explain large angle α-scatterings

Rutherford gave dynamic planetary model of atom in which nucleus is compared with sun and electrons are compared with planets





→ almost whole mass of the atom and +ve charge is concentrated at its centre called nucleus and –ve electrons revolve around nucleus in certain permitted orbits

→ es force of attraction between nucleus and electrons provides centripetal force required by the electrons to revolve around nucleus in circular orbits

→ explains large angle α-scatterings

→ diameter of nucleus: 10^{-14}m (by finding distance of closest approach) diameter of atom: 10^{-10}m $\Rightarrow$ nucleus is very small in size

CONSTITUENTS OF NUCLEUS:

Nucleus is small in size and is very dense.

PROTON-ELECTRON THEORY OF NUCLEUS

According to this theory nucleus consists of electrons and protons.

Favours:

1. Emission of electrons by radioactive nuclei during β-decay favours the existence of electrons inside them.
2. Masses of different nuclei are found to be whole number multiple of mass of hydrogen atom (i.e. proton) which favours existence of protons. Slight change in masses from whole numbers is due to the presence of various isotopes.

According to proton-electron theory, any nucleus $_ZX^A$ consists of A protons and A-Z electrons. In this way,

charge on nucleus Q = A(+e) + (A-Z)(-e) = +Ze and

mass of nucleus $m_N = A(m_p) + (A-Z)(m_e) \cong A\, m_p$ (as $m_e << m_p$)

$\cong$ A amu (as $m_p \cong$ 1amu)

In order that atom is electrically neutral, there must be Z extra nuclear electrons revolving around the nucleus.

Failures:

1. Wave mechanical considerations

If electrons exist inside the nucleus, maximum uncertainty in the measurement of their position is of the order of diameter of nucleus.

$$\Delta x = 2\times10^{-14} \text{ m}$$

From uncertainty principle, uncertainty in the measurement of momentum

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{2\times10^{-14}} = 0.527\times10^{-20} \text{ kgms}^{-1}$$

Magnitude of momentum possessed by electron should be at least equal to the uncertainty in its measurement.

$$p_{min} \sim \Delta p = 0.527\times10^{-20} \text{ kgms}^{-1}$$

Energy possessed by electron,

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

Now pc= 1.581×10^{-12}J and $m_0c^2 = 0.082\times10^{-12}$J << pc

$$\therefore E \cong pc = \frac{1.581\times10^{-12}}{1.6\times10^{-13}} \text{ MeV} \cong 10 \text{ MeV}$$

So if electron exists inside the nucleus, it must possess minimum energy of 10 MeV. But electrons coming out of nuclei in radioactive decays are found to have energy in the range 3-4 MeV. So electrons cannot exist inside the nucleus.

Even if we assume the existence of electrons of energy 3-4 MeV inside the nucleus, it will mean, lesser momentum and hence greater size of nucleus which is not possible. Hence electrons cannot exist inside the nucleus.

2. Nuclear spin and statistics

Experimental observation is that all even A nuclei have integral spin & obey BE statistics and all odd A nuclei have half integral spin & obey FD statistics.
Consider $_7N^{14}$ nucleus. According to proton-electron theory, it has 14 protons and 14-7=7 electrons inside it. So there are total of 21 particles of half integral spin and nucleus must have half integral spin and it must obey FD statistics. But experimentally it is found to have integral spin (even A) and obeys BE statistics. Hence electrons cannot exist inside the nucleus.

3. Nuclear magnetic moment

Nucleus is charged and rotation of charged particles gives rise to magnetic moment.
Magnetic moment of electron = $\frac{e\hbar}{2m_e} = \mu_B$
Magnetic moment of proton = $\frac{e\hbar}{2m_p} = \mu_N$

$\because \frac{m_p}{m_e} \cong 2000$

So magnetic moment of electron is much higher than that of proton. Expected magnetic moment of nucleus must be in units of Bohr's magneton.
But magnetic moment of various nuclei are found to be in units of nuclear magneton varying from $-2\mu_N$ to $+4\mu_N$. Hence electrons cannot exist inside the nucleus.

4. Finite size of the electron

In case of heavy nuclei, number of electrons in the nucleus is very large. Since electrons are spheres of finite dimensions, volume required to be occupied by these electrons become bigger than the volume of nucleus itself. So electrons cannot exist inside the nucleus.

5. Compton wavelength

A bound fundamental particle cannot be confined to a region smaller than its Compton wavelength. Since for electron $\lambda_c = \frac{h}{m_0 c} = 242\times10^{-14}$ m >> size of nucleus, so nucleus cannot accommodate electrons.

6. β^+-decay

If electron exists inside the nucleus β^+-decay cannot be explained. Because electron and positron annihilate together to produce γ-rays. So electron cannot exist inside the nucleus but come out from the nucleus at the time of β-decay in the process of change from one state of nucleon to another state. They do not exist permanently inside the nucleus but are created at the time of emission.

PROTON NEUTRON THEORY OF NUCLEUS

According to this theory, any nucleus $_ZX^A$ consists of Z protons and A-Z neutrons. In this way charge and mass of the nucleus are justified.

Q = Z(+e) + (A-Z)(0) = +Ze and mass of nucleus $m_N = Z(m_p) + (A-Z)(m_n) \cong A\, m_p$ (as $m_n \cong m_p$)

$\cong$ A amu (as $m_p \cong$ 1amu)

In order that atom is electrically neutral, there must be Z extra nuclear electrons revolving around the nucleus.

** proton and neutron are called two charged states of the same particle called nucleon.

** proton is called protonic state and neutron is called neutronic state of nucleon.

Favours:

1. Wave mechanical considerations

If neutron exists inside the nucleus, maximum uncertainty in the measurement of its position is of the order of diameter of nucleus.

$$\Delta x = 2\times10^{-14} \text{ m}$$

From uncertainty principle, uncertainty in the measurement of momentum

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{2\times10^{-14}} = 0.527\times10^{-20} \text{ kgms}^{-1}$$

If neutron exists inside the nucleus, magnitude of momentum possessed by it should be at least equal to the uncertainty in its measurement.

$$p_{min} \sim = \Delta p = 0.527\times10^{-20} \text{ kgms}^{-1}$$

Energy possessed by neutron/proton,

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

Now pc= 1.581×10^{-12}J and m_0c^2 = 150.3×10^{-12}J =939.375 MeV>> pc

$\therefore$ E $\cong m_0c^2 \cong$ 940 MeV which is almost same as experimental rest mass energy of proton or neutron i.e.939.6MeV. Neutron/proton with this much energy can exist inside the nucleus.

2. Nuclear spin and statistics

Experimental observation is that all even A nuclei have integral spin & obey BE statistics and all odd A nuclei have half integral spin & obey FD statistics.

Consider $_7N^{14}$ nucleus. According to proton-neutron theory, it has 7 protons and 14-7=7 neutrons inside it. So there are total of 14 particles of half integral spin and nucleus must have integral spin and it must obey BE statistics. Experimentally also, it is found to have integral spin (even A) and obeying BE statistics. Hence neutron/proton can exist inside nucleus.

3. Nuclear magnetic moment

Experimental magnetic moment of proton and neutron are found to be $+2.79\mu_N$ & $-1.91\mu_N$ respectively. Magnetic moment of nucleus must be in units of nuclear magneton if neutron/proton exist inside the nucleus. This agrees with experimental observation that various nuclei are found to have magnetic moment varying from $-2\mu_N$ to $+4\mu_N$. Hence neutron/proton can exist inside the nucleus.

4. Finite size

Since $m_p \cong m_n$ & total number of particles in the nucleus are A, so nucleus can easily accommodate (A-Z) neutrons in addition to Z protons.

5. Compton wavelength

A bound fundamental particle cannot be confined to a region smaller than its Compton wavelength. Since for neuton/proton $\lambda_c = \frac{h}{m_0 c} = 0.13\times10^{-14}$ m which is of the order of size of nucleus, so neutron/proton can exist inside the nucleus.

6. β^+-decay

Electrons do not preexist inside the nucleus but are produced at the time of emission by conversion of a nuclear proton into a neutron and vice-versa.

$$P \rightarrow n + e^+ + \nu_e \quad \& \quad n \rightarrow p + e^- + \overline{\nu_e}$$

PROPERTIES OF NUCLEUS

1. Nuclear mass

Mass of atom = mass of nucleus + Zm_e

$= m_N + Zm_e$

$\cong m_N \qquad (\because m_e << m_N)$

99.75% of atomic mass is nuclear mass.

→ nuclear mass is measured using mass spectrograph by noting e/m ratio and is found to be $\cong$ A amu

1 amu is defined as 1/12th of the mass of one ${}_6C^{12}$ atom.

$$1 \text{ amu} = \frac{1}{12} (\text{mass of } 1\ {}_6C^{12} \text{ atom})$$

$$= \frac{1}{12} \times \frac{12\ g}{6.02\times 10^{23}}$$

$$= 1.66 \times 10^{-24} \text{ g}$$

$$= 1.66 \times 10^{-27} \text{ kg}$$

In this way, $m_p \cong 1.672 \times 10^{-27}$ kg = 1.007277 amu

& $m_n \cong 1.675 \times 10^{-27}$ kg = 1.008665 amu

2. Nuclear energy

It is expressed in J, eV or MeV.

1 eV is defined as the energy acquired by an electron when it is accelerated through a potential difference of 1 volt.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1\text{V} \qquad (W=qV)$$

$$= 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$$

$$E = mc^2$$

$= (1 \text{ amu}) \times (9 \times 10^{16} \text{ m}^2/\text{s}^2)$

$= 1.66 \times 10^{-27}\text{kg} \times 9 \times 10^{16} \text{ m}^2/\text{s}^2$

$= 1.66 \times 9 \times 10^{-11} \text{ J}$

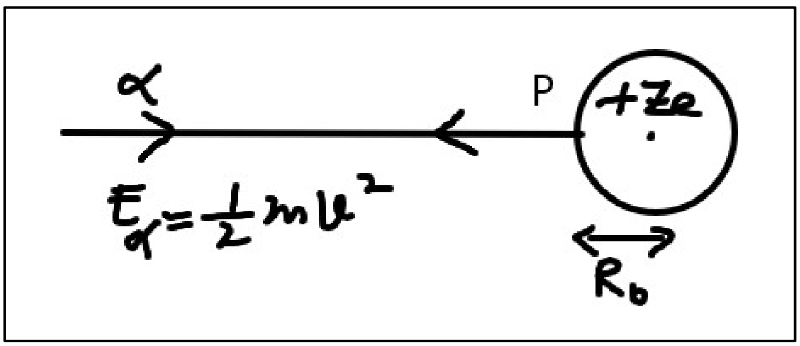
$= \frac{1.66 \times 9 \times 10^{-11}}{1.6 \times 10^{-13}} \text{ MeV}$

$= \frac{14.9}{1.6} \times 10^2 \text{ MeV} \cong 931 \text{ MeV}$

3. Nuclear Size

It is determined by scattering experiments of Rutherford.

Consider α-particles moving towards nucleus of charge +Ze. As it moves forward its kinetic energy decreases and potential energy increases. At P, whole of its K.E gets converted into P.E and particle returns back.



Distance of point P from centre of nucleus is termed as distance of closest approach.

$$\text{K.E.}, E = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0}\frac{(Ze)\times(2e)}{d}$$

$$\Rightarrow d = \frac{2Ze^2}{4\pi\epsilon_0 E}$$

For E= 8MeV = $8 \times 1.6 \times 10^{-13}$ J & Z = 79 (Gold nucleus),

$$d = \frac{2 \times 79 \times (1.6 \times 10^{-19})^2}{8 \times 1.6 \times 10^{-13}} \times 9 \times 10^9$$

$$= 2.84 \times 10^{-14} \text{ m}$$

Upper limit of d for gold nucleus = 3.2×10^{-14} m

for silver nucleus = 2×10^{-14} m

for helium nucleus = 3×10^{-15} m

Nucleus is considered almost spherical in shape.

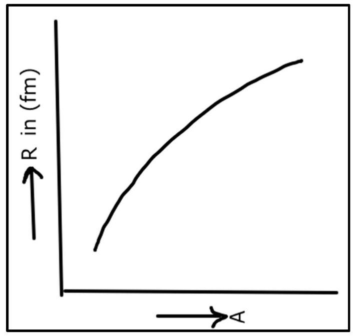
Volume of nucleus α number of particles i.e. A

$V \propto A$

$\frac{4}{3}\pi R^3 \propto A$

$R \propto A^{1/3}$

$R = R_0 A^{1/3}$



$R_0 = 1.2\text{-}1.6$ fm; average value of $R_0 = 1.4$ fm

4. <u>Nuclear Density</u>

Nucleus is small in size but highly dense. Consider a nucleus $_ZX^A$. It has A nucleons inside it.

Mass of each nucleon $= 1.67 \times 10^{-27}$kg

Mass of A nucleons $= A \times 1.67 \times 10^{-27}$kg

Volume of nucleus $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$

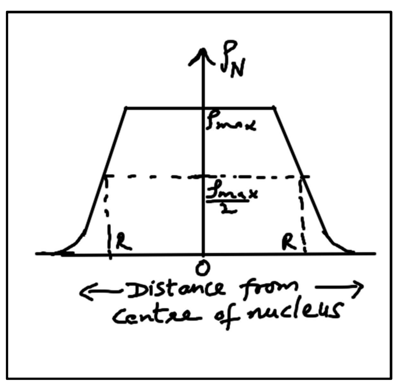
Density, $\rho = \frac{M}{V} = \frac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi R_0^3 A} = \frac{3 \times 1.67 \times 10^{-27}}{4\pi R_0^3} \neq f^n(A)$

$\Rightarrow$ density of nucleus is independent of mass number i.e. same for every nucleus.

For $R_0 = 1.2 \times 10^{-15}$ m

$\rho = 2.29 \times 10^{17}$ kg/m^3

Experimentally nuclear density is found to be nonuniform, being maximum at the centre of nucleus and decreasing towards its boundary.



Nuclear radius is defined as the distance from the centre of nucleus at which density becomes one half of its maximum value.

Radius of atom $\cong 10^{-10}$m

$\frac{density\ of\ atom}{density\ of\ nucleus} = \frac{m_A}{V_A} \times \frac{V_N}{m_N} \cong \frac{V_N}{V_A}$ (as $m_A \cong m_N$)

$= \frac{R_N^3}{R_A^3} = \left(\frac{10^{-1}}{10^{-10}}\right)^3 = 10^{-12}$

Density of atom = $10^{-12} \times 2.29 \times 10^{17}$

$= 2.29 \times 10^5$ kg/m^3

5. Nuclear charge

It is determined by adding charges on nucleons algebraically.

Q = Z(+e) + (A-Z)(0)

= + Ze

6. Wave Mechanical Properties

(i) Parity

Parity of a system refers to its behavior under reflection of coordinates.

x→-x, y→-y, z→-z

If Ψ(-x,-y,-z) = Ψ(x,y,z) system has +ve or even parity

If Ψ(-x,-y,-z) = - Ψ(x,y,z) system has –ve or odd parity

In general, Ψ(-x,-y,-z) = PΨ(x,y,z) where P = $\pm$ 1

For H-like atoms P = $(-1)^l$ where l is orbital quantum number.

For a system having,

- any number of even parity particles and even number of odd parity particles, parity is even.
- any number of even parity particles and odd number of odd parity particles, parity is odd.

(ii) Statistics

Statistics followed by a system is decided on the basis of behavior of wave function when coordinates of any two identical particles are interchanged.

- If interchanging the coordinates of any two identical particles leaves the wave function unchanged, it is called symmetric wave function and system obeys BE statistics.

  $\Psi(x_1, x_2, \ldots, x_j, \ldots, x_i, \ldots, x_n) = \Psi(x_1, x_2, \ldots, x_i, \ldots, x_j, \ldots, x_n)$
- If interchanging the coordinates of any two identical particles changes the sign of the wave function, it is called antisymmetric wave function and system obeys FD statistics.

  $\Psi(x_1, x_2, \ldots, x_j, \ldots, x_i, \ldots, x_n) = -\Psi(x_1, x_2, \ldots, x_i, \ldots, x_j, \ldots, x_n)$

7. Angular Momentum

Each nucleon in nucleus possesses angular momentum j which is sum of orbital angular momentum (ℓ=n$\hbar$) due to its motion about centre of nucleus and spin angular momentum (s= $\hbar/2$) due to its spinning motion about an axis passing through its centre of mass.

$\therefore$ j of nucleon = $\ell \pm$ s

Angular momentum of nucleus or nuclear spin, J = $\Sigma \ell \pm \Sigma s$

= L $\pm$ S for LS coupling

and J = Σ j = Σ ($\ell \pm$ s) for jj coupling

→ Nuclear spin in ground state < Nuclear spin in excited state

→ In even-even nuclei where Z and N are even, angular momentum is zero because of complete pairing.

→ In even-odd or odd-even nuclei, half integral spin angular momentum of single extra nucleon combines with integral orbital angular momentum to give half integral total angular momentum. Experimentally, angular momentum of such nuclei are found to lie between $\hbar/2$ to $9\hbar/2$.

→ In odd-odd nuclei, half integral spin angular momentum of single extra proton combines with half integral spin angular momentum of single extra neutron and integral orbital angular momentum to give integral total angular momentum in G.S.

So total angular momentum/nuclear spin is n$\hbar$ if nucleus is even-even or odd-odd (i.e. A is even) and (2n+1)$\hbar/2$ if nucleus is even-odd or odd-even (i.e. A is odd)

8. Magnetic Moment

Nucleons inside the nucleus are not stationary. They possess orbital as well as spin motion. Moving charge constitutes current and hence nucleus possesses magnetic moment. Magnetic moment of a plane current loop of radius r and current i due to charge +e is given by,

$\overrightarrow{\mu_l}$ = i $\vec{A}$

= i$\pi r^2\hat{r}$

= $\frac{+e}{T}\pi r^2\hat{r}$

= $\frac{+ev}{2\pi r}\pi r^2\hat{r}$

= $\frac{evrm_p\hat{r}}{2m_p} = \frac{e\vec{L}}{2m_p}$

Gyromagnetic motion for orbital motion is defined as the ratio of magnetic moment to the angular momentum,

$\gamma = \frac{\vec{\mu}_l}{\vec{L}} = \frac{e\vec{L}}{2m_p\vec{L}}$

Magnetic dipole moment along Z-direction,

$(\mu_l)_z = \frac{e}{2m_p}(L_z) = \frac{e}{2m_p}(m_l\hbar) = \frac{e\hbar}{2m_p}(m_l)$ where $m_l = -l\ through\ 0\ to + l$

This is classical relation. Quantum mechanics and experimental data shows that there must be a dimensionless correction factor called g-factor in the above relations i.e.

$\vec{\mu}_l = g_l\frac{e}{2m_p}\vec{L}$ and

$(\mu_l)_z = g_l \left(\frac{e\hbar}{2m_p}\right) m_l$

Similarly due to spin motion,

$\vec{\mu}_s = g_s \frac{e}{2m_p} \vec{S}$ and

$(\mu_s)_z = g_s \left(\frac{e\hbar}{2m_p}\right) m_s$ where $m_s = \pm\frac{1}{2}$

Total magnetic moment,

$\vec{\mu} = \overrightarrow{\mu_l} + \overrightarrow{\mu_s}$

$= (g_l \vec{L} + g_s \vec{S})\frac{e}{2m_p}$

$= g \frac{e}{2m_p} \vec{J}$ where $g = \frac{1}{2}(g_l + g_s) + \frac{1}{2}(g_l - g_s)\frac{L(L+1)-S(S+1)}{J(J+1)}$

The magnetic dipole moment of proton and neutron are found to be

$\therefore \mu_p = +2.79 \frac{e\hbar}{2m_p} = +2.79\ \mu_N$

& $\mu_n = -1.91 \frac{e\hbar}{2m_p} = -1.91\ \mu_N$

Expected magnetic moment of neutron is zero since it is uncharged. Actually, it possesses negative magnetic moment as it is supposed to consist of equal amount of +ve and –ve charges with –ve charge at higher distance from axis of rotation.

For e.g. take the case of deuteron $_1H^2$ nucleus.

It has one proton and one neutron.

$\therefore$ expected magnetic moment = $\mu_p + \mu_n$ = 2.792-1.913 = 0.879 μ_N which agrees with experimental value 0.858 μ_N .

9. <u>Electric Quadrupole Moment</u>

It measures departure of nucleus from spherically symmetric charge distribution.

Nuclei are not always spherical. In a spherical nucleus CM & CC coincide and dipole moment is zero.

Consider a charge +e located at A(x,y,z). Potential at P due to charge at A,

$V = \frac{1}{4\pi\epsilon_0}\frac{e}{R_0}$

Now $\vec{r} + \overrightarrow{R_0} = \vec{R}$

$\Rightarrow \overrightarrow{R_0} = \vec{R} - \vec{r}$

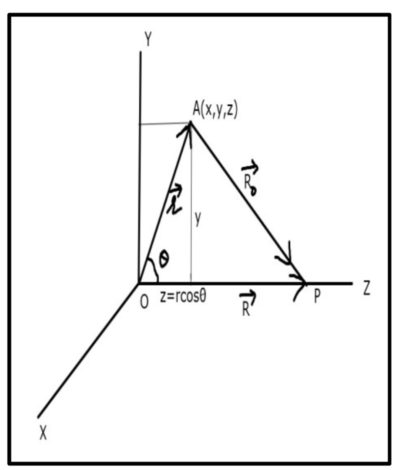
$\Rightarrow \overrightarrow{R_0} \cdot \overrightarrow{R_0} = (\vec{R} - \vec{r}) \cdot (\vec{R} - \vec{r})$

$\Rightarrow R_0^2 = R^2 + r^2 - 2\vec{R} \cdot \vec{r}$

$\Rightarrow R_0 = \left\{R^2 + r^2 - 2\vec{R} \cdot \vec{r}\right\}^{1/2}$

$= \{R^2 + r^2 - 2Rr\cos\theta\}^{1/2}$

$\therefore V = \frac{e}{4\pi\epsilon_0}\{R^2 + r^2 - 2Rr\cos\theta\}^{-1/2}$



$$= \frac{e}{4\pi\epsilon_0 R}\left\{1 + \frac{r^2}{R^2} - \frac{2r\cos\theta}{R}\right\}^{-1/2}$$

$$= \frac{e}{4\pi\epsilon_0 R}\left[1 + \left(\frac{-1}{2}\right)\left(\frac{r^2}{R^2} - \frac{2r\cos\theta}{R}\right) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{2!}\left(\frac{r^2}{R^2} - \frac{2r\cos\theta}{R}\right)^2 + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)}{3!}\left(\frac{r^2}{R^2} - \frac{2r\cos\theta}{R}\right)^3 + \ldots..\right]$$

$$= \frac{e}{4\pi\epsilon_0 R}\left[1 + \frac{1}{2}\cdot\frac{2rc}{R} + \frac{r^2}{R^2}\left(\frac{-1}{2} + \frac{3}{2}\cos^2\theta\right) + \frac{r^3}{R^3}\left(\frac{-3}{2}\cos\theta + \frac{5}{2}\cos^3\theta\right) + \ldots\ldots\right]$$

Coefficient of $\frac{1}{R}$ is called monopole moment,

Coefficient of $\frac{1}{R^2}$ is called dipole moment,

Coefficient of $\frac{1}{R^3}$ is called quadrupole moment & so on.

$$Q = \frac{er^2}{4\pi\epsilon_0}\left(\frac{-1}{2} + \frac{3}{2}\cos^2\theta\right)$$

$$= \frac{er^2}{8\pi\epsilon_0}(3\cos^2\theta - 1)$$

$$= \frac{er^2}{8\pi\epsilon_0}\left(3\frac{z^2}{r^2} - 1\right)$$

$$= \frac{e}{8\pi\epsilon_0}(3z^2 - r^2)$$

For spherically symmetric nucleus, x=y=z

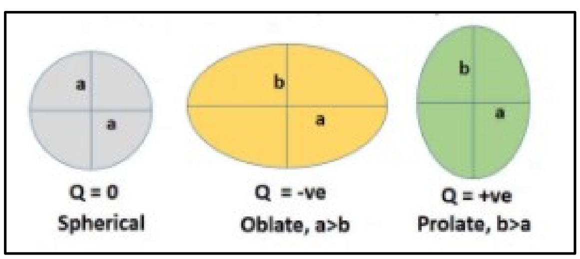
$r^2 = x^2 + y^2 + z^2 = 3z^2$

$\therefore$ Q = 0

$Q \neq 0$ when protons are placed asymmetrically.

Actually for Z protons, nucleus is considered as an ellipsoid of revolution with diameter 2b along the axis of symmetry and 2a along an axis perpendicular to the axis of symmetry.

$$Q = \frac{2}{5}(b^2 - a^2)(Ze)$$



It is measured in barns where 1 barn = 10^{-28} m^2 i.e. it is of the order of nuclear area.

10. Classification Of Nuclei

(i) Isotopes

These are nuclei of elements having same Z but different A.

e.g. ${}_8O^{16}$, ${}_8O^{17}$, ${}_8O^{18}$

* They occupy same position in periodic table.
* No. of protons are same but no. of neutrons are different.

* They have same chemical properties.

* They have different nuclear properties like $_{11}Na^{23}$ is stable, $_{11}Na^{24}$ attains stability by undergoing β^--decay and $_{11}Na^{22}$ attains stability by undergoing β^+-decay.

(ii) Isobars

These are nuclei of elements having same A but different Z.

e.g. $_{18}Ar^{40}$, $_{20}Ca^{40}$

* They occupy different position in periodic table.

* Total no. of particles are same.

*They have different chemical properties.

(iii) Isotones

These are nuclei of elements having same number of neutrons i.e. N=A-Z

e.g. $_{6}C^{13}$, $_{7}N^{14}$

(iv) Isomers

These are nuclei of elements having same Z and A but differ in their nuclear energy states, life times and internal structure.

(v) Mirror nuclei

These are nuclei of elements in which number of number of protons and number of neutrons are interchanged.

e.g. $_{4}Be_{3}^{7}$, $_{3}Li_{4}^{7}$

(vi) Isodiaspheres

These are nuclei of elements having same D i.e. excess of neutrons over protons

i.e. same D=N-Z=(A-Z)-Z=A-2Z

e.g. $_{6}C_{7}^{13}$, $_{1}H_{2}^{3}$

NUCLEAR FORCES

According to neutron-proton theory of nucleus, inside the nucleus there are protons and neutrons. These positively charged protons tend to disrupt the nucleus because of forces of repulsion between them. The force binding the nucleons together inside the nucleus cannot be gravitational in nature as its magnitude is very small as compared to es force. So there must be some other kind of force. Yukawa said that like gravitons are exchange particles for gravitational forces, photons are exchange particles for es forces, in a similar way some particles must be exchanged amongst nucleons. He predicted that particles exchanged between nucleons must be around 200 times heavier than electrons. Anderson experimentally discovered π-mesons with mass$\cong$270m_e in accordance with Yukawa's theory.

According to this theory,

1. All nucleons consist of some common core surrounded by pulsating cloud of π-mesons which are three in number π^+, π^- and π^0. These nucleons are so close that their meson clouds overlap.
2. Π-mesons remain free for such a short interval of time (10^{-23}s) that they are undetected and are called virtual mesons. So π-meson clouds surrounding the nucleons are continuously created and annihilated.
3. A π-meson can be absorbed not only by its own nucleon, but also by some other nucleons if it is in the meson cloud of the latter. This transfer of the meson from one nucleon to another is responsible for nuclear interaction.
4. The force between proton and neutron is because of exchange of charged mesons whereas the force between like nucleons is due to exchange of neutral mesons (dog and bone analogy)

   $n \leftrightarrow p + \pi^-$

   $p \leftrightarrow n + \pi^+$

   $n \leftrightarrow n' + \pi^0$

   $p \leftrightarrow p' + \pi^0$
5. The meson travels with speed of light and returns to the nucleon within time Δt where $\Delta t = \frac{h}{2\pi\Delta E} = \frac{h}{2\pi m_\pi c^2}$.
6. The exchange of energy between nucleons is allowed for time Δt.
7. The distance within which the exchange of mesons by nucleons takes place is thr range of nuclear force and is given by

   Range = $c\Delta t = \frac{h}{2\pi m_\pi c} = 1.4 \times 10^{-15}$m with $m_\pi = 140\text{MeV}/c^2$
8. Taking range of nuclear force as 1.5fm, Yukawa estimated the mass of meson as

   $m_\pi c^2 = \frac{hc}{2\pi \times Range} = 150$ MeV

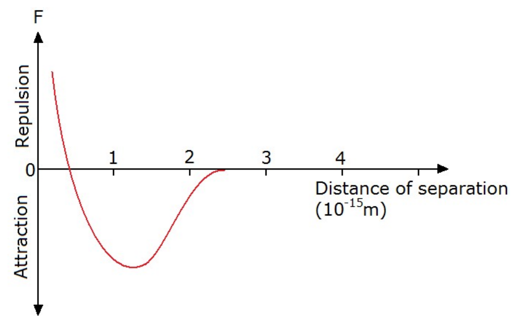
Properties of nuclear forces:

1. Strongest forces

Nuclear force is the strongest force in nature. It is 100 times stronger than the electromagnetic force (which binds electrons into atoms), 10000 times stronger than the weak force (which governs radioactive decay) and 10^{38} times stronger than gravitational force (which attracts you to the earth and earth to the sun). For e.g. the binding energy of deuteron is 2.23MeV whereas binding energy of hydrogen atom which is due to es forces is only 13.6eV.

2. Short range forces

The nuclear force is powerfully attractive at distances of about 1fm but decreases rapidly and becomes negligible for distances beyond 2.5fm. At distances less than 0.7fm, it becomes repulsive. So nucleons cannot come closer than this distance. This repulsive component is responsible for the physical size of nuclei and prevents collapse of nucleus.



3. Saturated forces

Nuclear forces are saturated forces as binding energy per nucleon is almost constant (A>40). This means that once a nucleon has formed a few bonds, it ignores any further nucleons that may be added to the system i.e. nucleons exhibit nn interactions only. To justify this, let us suppose that nuclear forces are unsaturated forces. So each nucleon can interact with all other A-1 nucleons. So there will be total of ${}^{A}C_2 = \frac{A(A-1)}{2!}$ Pairs of nucleons and B.E. is proportional to A^2 which is against experimental observation that B.E. is proportional to A. Thus our supposition is wrong. Hence nuclear forces are saturated forces.

4. Spin dependent force

It has been observed that the deuteron (the proton-neutron bound state, the smallest nucleus) deviates slightly from a spherical shape and has a nonzero quadrupole moment. This suggests a force that depends upon the orientation of spins of the nucleons with regard to the vector joining them.

Nuclear force between two nucleons having parallel spins is found to be stronger than the force between two nucleons having antiparallel spin. Thus nuclear forces are spin dependent forces.

5. Exchange forces

Nuclear forces are called exchange forces as they are observed because of exchange of π-mesons between nucleons. Nucleons are able to exchange charges, spin projections and coordinates because of exchange property of nuclear forces.

6. Charge independent

Nuclear forces are charge independent forces. This means that the force between two protons, two neutrons and a proton & a neutron are nearly the same when em forces are ignored.

For e.g. in mirror nuclei ${}_1H^3$ (1p and 2n) and ${}_2He^3$ (2pand 1n), B.E. of ${}_1H^3$ (due to two n-p and one n-n bond) is found to be 8.5MeV which is higher than 7.7MeV for ${}_2He^3$ (due to two n-p and one p-p bond). This difference is due to es force of repulsion between protons. If allowance is made for this force, the n-n & p-p forces are equal. Hence nuclear forces are charge independent forces.

7. Non central forces

Nuclear forces are non central forces i.e. they do not act along the line joining two particles.

MASS DEFECT

It is defined as the difference between sum of masses of constituents and mass of nucleus. Consider a nucleus ${}_ZX^A$. Let m_p, m_n and m_N be the masses of proton, neutron and nucleus respectively.

Sum of masses of constituents of nucleus = $Zm_p + (A-Z)m_n > m_N$

$$\Delta m = Zm_p + (A-Z)m_n - m_N$$

PACKING FRACTION

It is a term that shows deviation of atomic masses from whole numbers.

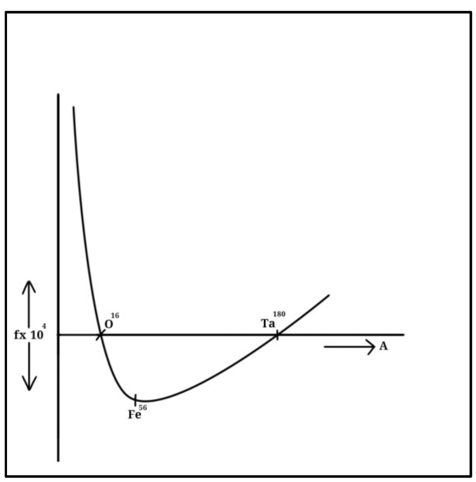
$f = \frac{M-A}{A}$ where M is atomic mass and A is mass number.

Now $M = m_N + Zm_e \cong m_N$

$Zm_p + (A-Z)m_n = Z + A - Z$ (as $m_p \cong m_n \cong 1$amu)

$= A$

$\therefore f = \frac{m_N - A}{A} = \frac{\Delta m}{A}$ (ignoring –ve sign)



Hence packing fraction can be expressed as mass defect per nucleon.

(i) For very light nuclei packing fraction is high and positive, so these nuclei are less stable.

(ii) As mass number increases, f decreases and becomes zero for O^{16}.

(iii) With further increase in A, f decreases and becomes minimum for Fe^{56} which is most stable nucleus.

(iv) As A increases beyond 56, f increases and becomes zero again for Ta^{180}(Tantalum).

(v) Further increase in A leads to increasing values of f. So heavy nuclei are also unstable.

(vi) Tendency of nucleus is to shift from higher f region to lower f region. This is possible by nuclear fusion in case of lighter nuclei and by nuclear fission in case of heavier nuclei.

(vii) –ve packing fraction corresponds to exceptional stability i.e. when nucleus is formed, there is decrease in mass which is released as energy. Same amount of energy must be supplied to the nucleus in order to breakup into its costituents.

BINDING ENERGY

The energy equivalent to mass defect is called binding energy. It is the amount of energy needed to bind the nucleons inside the nucleus.

$$B.E. = \Delta mc^2$$
$$= [Zm_p + (A\text{-}Z)m_n - m_N]c^2$$

This is the energy required to break the nucleus into individual nucleons and keep them at infinite distance apart.

B.E. per nucleon is the energy required to extract one nucleon from the nucleus.

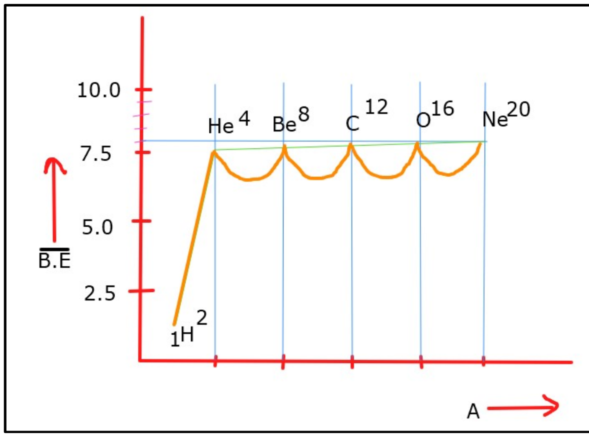
$$\text{Now B.E.} = [Zm_p + (A\text{-}Z)m_n - m_N]c^2$$
$$= [Zm_p + Zm_e + (A\text{-}Z)m_n - m_N - Zm_e]c^2$$
$$= [Zm_H + (A\text{-}Z)m_n - M]c^2$$
$$= [Am_N - Z(m_n - m_H) - M]c^2$$
$$\therefore \overline{B.E.} = \frac{B.E.}{A} = \left[m_n - \frac{Z}{A}(m_n - m_H) - \frac{M}{A}\right]c^2$$
$$= \left[(m_n - 1) - \frac{Z}{A}(m_n - m_H) - \frac{M}{A} + 1\right]c^2$$
$$= \left[(m_n - 1) - \frac{Z}{A}(m_n - m_H) - f\right]c^2$$

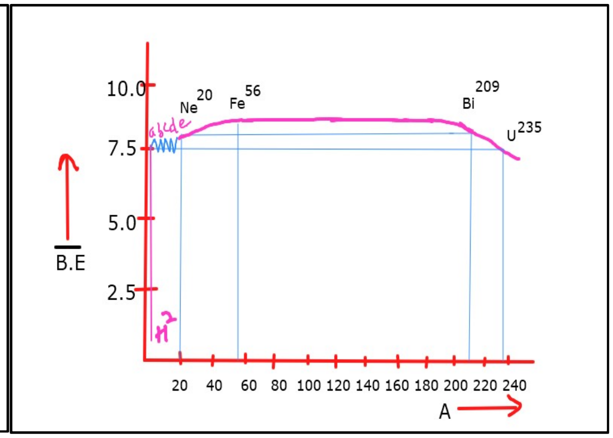
For moderate nuclei, $40 \leq A \leq 120$,

$\frac{Z}{A}$: 0.5 to 0.42 and f = -6×10^{-4}amu/nucleon

$m_n - m_H = 0.000840$ amu

$\frac{B}{A}$ = 8.5MeV/nucleon





(i) B.E. per nucleon for light nuclei like H^2, He^3 is very low, so these nuclei are less stable.

(ii) For mass numbers ranging from 2 to 20, there are sharply defined peaks corresponding to He^4, Be^8, C^{12}, O^{16} & Ne^{20}. These peaks show extra stability of these nuclei than their neighbours.

(iii) The curve has a broad maximum in mass number range 40 to 120 with highest $\overline{B.E.}$ of 8.8 MeV for Fe^{56}. So Fe^{56} is the most stable nucleus.

(iv) Ne^{20} and Bi^{209} have nearly same binding energy per nucleon.

(v) For A > 56, $\overline{B.E.}$ decreases and for U^{235} it is very less 7.5MeV which shows lesser stability of heavy nuclei.

(vi) Tendency of nucleus is to shift from lesser average binding energy region to higher average binding energy region. This is possible by nuclear fusion in case of lighter nuclei and by nuclear fission in case of heavier nuclei.